Benchmark Example No. 12

Cantilever in Torsion

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The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
1 Problem Description

The problem consists of a cantilever beam as shown in Fig. 1. The tip of the cantilever is offsetted in $y$-direction by $\Delta_y = l/200 = 2.5 \text{ cm}$, creating a geometrical imperfection. The beam is loaded with a transverse force $P_z$ and an axial force $P_x$. The imperfection acts as a lever arm for the loading, causing a torsional moment. The torsional moment at the support with respect to the local and global coordinate system is determined.

![Figure 1: Problem Description](image)

2 Reference Solution

In order to account for the effect of the geometrical imperfection on the structure, second-order theory should be used, where the equilibrium is established at the deformed system. According to the equilibrium of moments at the deformed system, with respect to the global $x$-axis, the torsional moment at the support $M_{x_{\text{global}}}$ is:

$$M_{x_{\text{global}}} = P_x (u_y + \Delta_y) - P_y u_z ,$$  \hfill (1)

whereas by the local $x$-axis the torsional moment $M_{x_{\text{local}}}$ is:

$$M_{x_{\text{local}}} = P_x u_y + P_x \left(\frac{\Delta_y}{l}\right) u_z ,$$  \hfill (2)

where $l$ is the length of the beam, $\Delta_y$ the initial geometrical imperfection and $P_x$ is negative for compression.
3 Model and Results

The properties of the model [1] [2] are defined in Table 1. A standard steel material is used as well as a standard hot formed hollow section with properties according to DIN 59410, DIN EN 10210-2. A safety factor $\gamma_M = 1.1$ is used, which according to DIN 18800-2 it is applied both to the yield strength and the stiffness. Furthermore, the self weight, the shear deformations and the warping modulus $C_M$ are neglected. At the support the warping is not constrained.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S\ 355$</td>
<td>$l = 5\ m$</td>
<td>$P_z = 10\ kN$</td>
</tr>
<tr>
<td>$\gamma_M = 1.1$</td>
<td>$RRo/SH\ 200 \times 100 \times 10$ [3]</td>
<td>$P_x = 100\ kN$</td>
</tr>
<tr>
<td>$C_M = 0$</td>
<td>$\Delta_y = 2.5\ cm$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>$u_y$</th>
<th>$u_z$</th>
<th>$M_{global}$</th>
<th>$M_{local}$</th>
<th>$P_{buck}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[cm]</td>
<td>[cm]</td>
<td>[kNcm]</td>
<td>[kNcm]</td>
<td>[kN]</td>
</tr>
<tr>
<td>SOF.</td>
<td>3.209</td>
<td>10.204</td>
<td>57.08</td>
<td>26.98</td>
<td>163.7</td>
</tr>
<tr>
<td>Ref.[4]</td>
<td>3.20</td>
<td>10.2</td>
<td>57.0</td>
<td>26.9</td>
<td>164</td>
</tr>
</tbody>
</table>

The corresponding results are presented in Table 2. Figure 2 shows the deformed shape of the structure and the nodal displacements for the $z$ and $y$ direction. From the presented results, we can observe that the values of the moments are correctly computed. Here has to be noted that the reference results are according to [1], where they are computed with another finite element software, and not with respect to an analytical solution.
4 Conclusion

This example presents a case where torsion is induced to the system because of an initial geometrical imperfection. It has been shown that the behaviour of the beam is captured accurately.

5 Literature