Benchmark Example No. 18

Three-storey Column under Large Compressive Force and Torsional Moment
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The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
Overview

Element Type(s): B3D
Analysis Type(s): STAT, GNL
Procedure(s): 
Topic(s): 
Module(s): ASE, DYNA
Input file(s): three_storey_column.dat

1 Problem Description

The problem consists of a three-storey column, subjected to a large compressive axial force $N$ and a torsional moment $M_T$ at the middle, as shown in Fig. 1. The rotation and twisting as well as the torsional moments of the structure are determined.

![Figure 1: Problem Description](image)

2 Reference Solution

A large axial compressive force is applied to the column of Fig. 1, in combination with a torsional moment at the middle, which can cause warping and potentially buckling of the structure. In order to account for this effect, second order theory has to be utilised. The total torsional moment $M_T$ is given as a sum of the different torsional parts, the primary, secondary and third respectively:

$$
\sum M_x = M_T = M_{T1} + M_{T2} + M_{T3},
$$

(1)

where

$$
M_{T1} = G I_T \phi',
$$

(2)
Three-storey Column under Large Compressive Force and Torsional Moment

\[ M_{T2} = -EC_M \phi''' , \quad (3) \]

\[ M_{T3} = N i_p^2 \phi' , \quad (4) \]

and the warping moment

\[ M_\omega = -EC_M \phi'' . \quad (5) \]

where \( G \) is the shear modulus, \( I_T \) the torsional moment of inertia, \( i_p \) the polar radius of gyration and \( EC_M \) the warping torsion stiffness. Introducing the above into Eq. 1 we have:

\[ (GI_T + N i_p^2) \phi' - EC_M \phi''' = M_T = \sum M_x . \quad (6) \]

3 Model and Results

The properties of the model [1] are defined in Table 1. A standard steel material is used and an I-beam profile for the cross-section. A safety factor \( \gamma_M = 1.1 \) is used, which according to DIN 18800-2 it is applied both to the yield strength and the stiffness. At the supports the warping is not constrained. The cross-sectional properties given in Table 1 are the values calculated by SOFiSTiK, matching the analytical solution, except from the torsional moment \( I_T \) and the warping modulus \( C_M \) which are modified to match the values of the reference example. This modification is done only for the sake of comparison and it has to be noted that the reference results [1] are computed with another finite element software, and not with respect to an analytical solution.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_M = 1.1 )</td>
<td>( b = 180 \text{ mm} )</td>
<td>( N = 1712 \text{ kN} )</td>
</tr>
<tr>
<td>( l = 6 \text{ m} )</td>
<td>( h = 400 \text{ mm} )</td>
<td>( M_t = 272 \text{ kN cm} )</td>
</tr>
<tr>
<td>S 355</td>
<td>( t_{\text{web}} = 10 \text{ mm} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( t_{\text{flange}} = 14 \text{ mm} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( C_M = 506884 \text{ cm}^6 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( I_y = 23071.6 \text{ cm}^4 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( I_z = 1363.9 \text{ cm}^4 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( I_T = 44.18 \text{ cm}^4 )</td>
<td></td>
</tr>
</tbody>
</table>

The results are presented in Table 2 and Fig 2. The value of \( M_{T3} \) is not given in the Reference [1], but
According to Eq. 4, the torque is computed as $-721\, kNcm$, which matches the calculated value by SOFiSTiK. If we now sum the torsional moment parts, it is observed that Eq. 1 is satisfied and that the total torsional moment at $x = 0$ is $136\, kNcm$.

### Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>SOF.</th>
<th>Ref.[1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi ,[mrad] ,(x = l/2)$</td>
<td>294.4</td>
<td>294</td>
</tr>
<tr>
<td>$\phi' ,[mrad/cm] ,(x = 0; x = l)$</td>
<td>1.5096</td>
<td>1.50965</td>
</tr>
<tr>
<td>$M_{T1} ,[kN,cm] ,(x = 0)$</td>
<td>491</td>
<td>491</td>
</tr>
<tr>
<td>$M_{T2} ,[kN,cm] ,(x = 0)$</td>
<td>366</td>
<td>366</td>
</tr>
<tr>
<td>$M_{T3} ,[kN,cm] ,(x = 0)$</td>
<td>$-721$</td>
<td>-</td>
</tr>
<tr>
<td>$M_T ,[kN,cm] ,(x = 0)$</td>
<td>136</td>
<td>136</td>
</tr>
<tr>
<td>$M_{\omega} ,[kN,cm]$</td>
<td>85583</td>
<td>85620</td>
</tr>
</tbody>
</table>

![Figure 2: Results [kN m]](image)

### 4 Conclusion

This example examines the torsional behaviour of the beam and the different parts involved in the calculation of the total torsional moment. The results are reproduced accurately.

### 5 Literature

Three-storey Column under Large Compressive Force and Torsional Moment