Benchmark Example No. 29

Cylindrical Hole in an Infinite Mohr-Coulomb Medium
1 Problem Description

This problem verifies stresses for the case of a cylindrical hole in an infinite elastic-plastic medium subjected to a constant in-situ state, as shown in Fig. 1. The material is assumed to be linearly elastic and perfectly plastic with a failure surface defined by the Mohr-Coulomb criterion. The stresses and the displacements are verified.

![Figure 1: Problem Description](image)

2 Reference Solution

Consider a hollow cylinder with inner radius \( \alpha \) and outer radius \( r \), under plane strain conditions, with a uniform pressure applied to its outer surface. If this pressure is slowly increased from 0 to some value \( P_o \), at first the cylinder will everywhere be in the elastic zone. As \( P_o \) increases further, the yielding will start, the yielded zone will grow radially outward, and the cylinder will consist of an inner annular region that has yielded and an outer annulus that is still in its elastic state [1]. A specialised problem is now the calculation of the stresses outside a cylindrical hole in an infinite elastic-perfectly-plastic medium, here with a failure surface defined by the Mohr-Coulomb criterion. Assume that the rock mass is initially under hydrostatic stress \( P_o \) and then a circular hole of radius \( \alpha \) is drilled into the rock, so that the stress at \( r = \alpha \) is reduced to some value \( P_i \). The yield zone radius \( R_o \) is given analytically by the theoretical model based on the solution of Salencon [2]

\[
R_o = \alpha \left( \frac{2 \left( P_o + \frac{q}{K_p-1} \right)}{\left( K_p + 1 \right) \left( P_i + \frac{q}{K_p-1} \right)^{\frac{1}{K_p-1}}} \right) \tag{1}
\]
where \( \alpha \) is the radius of the hole, \( P_o \) the initial in-situ stress, \( P_i \) the internal pressure and \( K_p, q \) are given by

\[
K_p = \frac{1 + \sin \phi}{1 - \sin \phi}
\] (2)

\[
q = 2c \tan(45 + \phi/2)
\] (3)

The parameters \( c \) and \( \phi \) correspond to the cohesion and angle of friction of the medium respectively. For sufficiently small values of \( P_o \), where \( P_o < P_i \) holds, the medium will be in its elastic state, and the stresses will be given by [1] [3]

\[
\sigma_r = P_o - (P_o - \sigma_{re}) \left(\frac{R_o}{r}\right)^2
\] (4)

\[
\sigma_\theta = P_o + (P_o - \sigma_{re}) \left(\frac{R_o}{r}\right)^2
\] (5)

where \( r \) is the distance from the field point to the center of the hole and \( \sigma_{re} \) is the radial stress at the elastic-plastic interface

\[
\sigma_{re} = \frac{1}{K_p + 1} (2P_o - q)
\] (6)

For \( P_o > P_i \), the rock will fail within some annular region surrounding the borehole. The stresses in the yielded zone will be given by

\[
\sigma_r = -\frac{q}{K_p - 1} + \left(\frac{P_i + \frac{q}{K_p - 1}}{\frac{r}{\alpha}}\right)^{K_p-1}
\] (7)

\[
\sigma_\theta = -\frac{q}{K_p - 1} + K_p \left(\frac{P_i + \frac{q}{K_p - 1}}{\frac{r}{\alpha}}\right)^{K_p-1}
\] (8)

3 Model and Results

The properties of the model are defined in Table 1. The radius of the hole is 1 m and is assumed to be small compared to the length of the cylinder, therefore 2D plane strain conditions are in effect. A fixed external boundary is located 29.7 m from the hole center. The model is presented in Fig. 2. The stresses are calculated and verified with respect to the formulas provided in Section 2.
Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Pressure Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = 6777.9 \text{ MPa}$</td>
<td>$\alpha = 1 \text{ m}$</td>
<td>$P_o = 30 \text{ MPa}$</td>
</tr>
<tr>
<td>$\nu = 0.21$</td>
<td>$r_{\text{boundary}} = 29.7 \text{ m}$</td>
<td>$P_i = 0$ or $1 \text{ MPa}$</td>
</tr>
</tbody>
</table>

Figure 2: Finite Element Model

Figure 3: Radial and Tangential Stresses for Cylindrical Hole in Infinite Mohr-Coulomb Medium

Figure 3 show the radial and tangential stress, along a line, lying on the X-axis. Results are presented for two cases, first with no internal pressure and second with $P_i = 1 \text{ MPa}$. The results in both cases are in very good agreement with the reference solution.
4 Conclusion

This example verifies the stresses of a cylindrical hole in an infinite elastic-perfectly-plastic medium. It has been shown that the behaviour of the model is captured accurately.

5 Literature

