Benchmark Example No. 30

Strip Loading on an Elastic Semi-Infinite Mass

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The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
1 Problem Description

This problem concerns the analysis of a strip loading on an elastic semi-infinite mass, as shown in Fig. 1. The material is assumed to be isotropic and elastic. The stresses are verified.

![Problem Description](image)

2 Reference Solution

The problem focuses on the calculation of the stresses due to a strip loading on an semi-infinite mass. The stresses under the surface are given by [1]:

\[
\sigma_y = \frac{p}{\pi} \left[ \alpha + \sin \alpha \cos (\alpha + 2\delta) \right]
\]

(1)

\[
\sigma_x = \frac{p}{\pi} \left[ \alpha - \sin \alpha \cos (\alpha + 2\delta) \right]
\]

(2)

and the principal stresses are

\[
\sigma_1 = \frac{p}{\pi} \left[ \alpha + \sin \alpha \right]
\]

(3)

\[
\sigma_3 = \frac{p}{\pi} \left[ \alpha - \sin \alpha \right]
\]

(4)
where $p$, $\alpha$, $\delta$ are described in Fig. 2

![Figure 2: Vertical Strip Loading on a Semi-Infinite Mass](image)

### 3 Model and Results

The properties of the model are defined in Table 1. The strip footing has a width of 2 m. The material is considered to be isotropic and elastic and plane strain conditions are in effect. For the analysis, boundary conditions are applied as shown in Fig. 3. The model is analysed with various dimensions in order to record the influence of the boundary in the results. The stresses are calculated and verified with respect to the formulas provided in Section 2. The results are printed for the case of a vertical line (cut) for $x = 0$ where the stresses in $x$ and $y$ coincide with the principal stresses.

![Figure 3: Finite Element Model](image)

**Table 1: Model Properties**

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Pressure Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = 20000 \text{ MPa}$</td>
<td>$H = 25, 50, 100 \text{ m}$</td>
<td>$P = 1 \text{ MPa/area}$</td>
</tr>
<tr>
<td>$\nu = 0.2$</td>
<td>$B = 2H$</td>
<td></td>
</tr>
</tbody>
</table>

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Fig. 4 shows the horizontal and vertical stress along the cutting line, for the analysed models with various dimensions. This line (cut) can be visualised in Fig. 5, where the contours of the vertical stress for the case of $H = 50 \text{ m}$ are illustrated. From the results of the stresses, it is evident that the vertical stresses are not influenced significantly from the dimensions of the model. On the contrary, for the horizontal stresses it is obvious, that as the boundary moves further away, its influence vanishes and the results are in very good agreement with the reference solution.
4 Conclusion

This example verifies the distribution of stresses of a semi-infinite mass under strip loading. It has been shown that the behaviour of the model is captured accurately.

5 Literature