Benchmark Example No. 31

Snap-Through Behaviour of a Truss

SOFiSTiK | 2018

VERiFiCATiON MANUAL
BE31: Snap-Through Behaviour of a Truss

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Front Cover

Project: New SOFiSTiK Office, Nuremberg | Contractor: WOLFF & MLLER, Stuttgart | Architecture: WABE-PLAN ARCHITEKTUR, Stuttgart | Structural Engineer: Boll und Partner. Beratende Ingenieure VBI, Stuttgart | MEP: GM Planen + Beraten, Griesheim | Lead Architect: Gerhard P. Wirth gpwirtharchitekten, Nuremberg | Vizualisation: Armin Dariz, BiMOTiON GmbH

Overview	
Element Type(s):	TRUS
Analysis Type(s):	STAT, GNL
Procedure(s):	LSTP
Topic(s):	
Module(s):	ASE
Input file(s):	snap_through.dat

1 Problem Description

This problem is concerned with one of the fundamental geometric non-linearity (GNL) tests. A simple two-node truss, as shown in Fig. 1, is examined in terms of the limit load and snap-through behaviour.

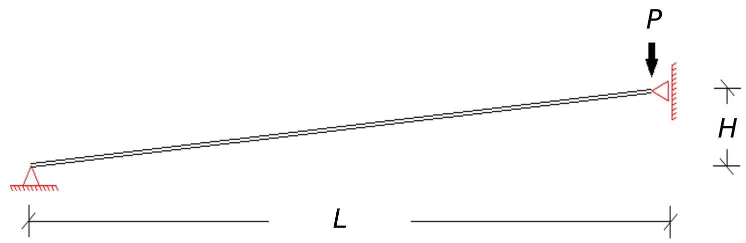

Figure 1: Problem Description

2 Reference Solution

In this problem a truss is pin-jointed to a rigid surface at one end and subjected to a transverse vertical point force at the other end as shown in Fig. 1. The loaded end is restrained to move only vertically and the truss in inclined with respect to the horizontal. This problem is effectively a symmetrical half of a two-bar structure and is utilised here in order to demonstrate the snap-through behaviour and limit points. The analytical solution assuming a shallow strut is given by [1] [2]

$$P = \frac{EAH^3}{2L^3}\left(\frac{2u}{H} + \frac{3u^2}{H^2} + \frac{u^3}{H^3}\right) \tag{1}$$

where the parameters H and L are shown in Fig. 1. The reference solution is plotted in Fig. 2.The load-displacement curve rises until it reaches a limit point A. If we continue further, the next point will be B, where the bar is horizontal and the vertical load reduces to zero. Further increments cause the bar to deflect below the horizontal axis until the second limit point is reached at point C. Note that after point B, the load reverses its sign and acts upwards. After point C, the bar continues its motion downwards until it reaches point D, where the vertical load is zero.

Note that under load-control approach, snap-through behaviour occurs after the first limit point A, where the bar suddenly jumps from point A to point E without any increase in the load. By switching from load-control to displacement-control, i.e. the displacement rather than the load is applied in small increments, the solution is able to progress beyond the limit point A, where further displacements cause the load to reduce as the bar reaches a horizontal position at B.

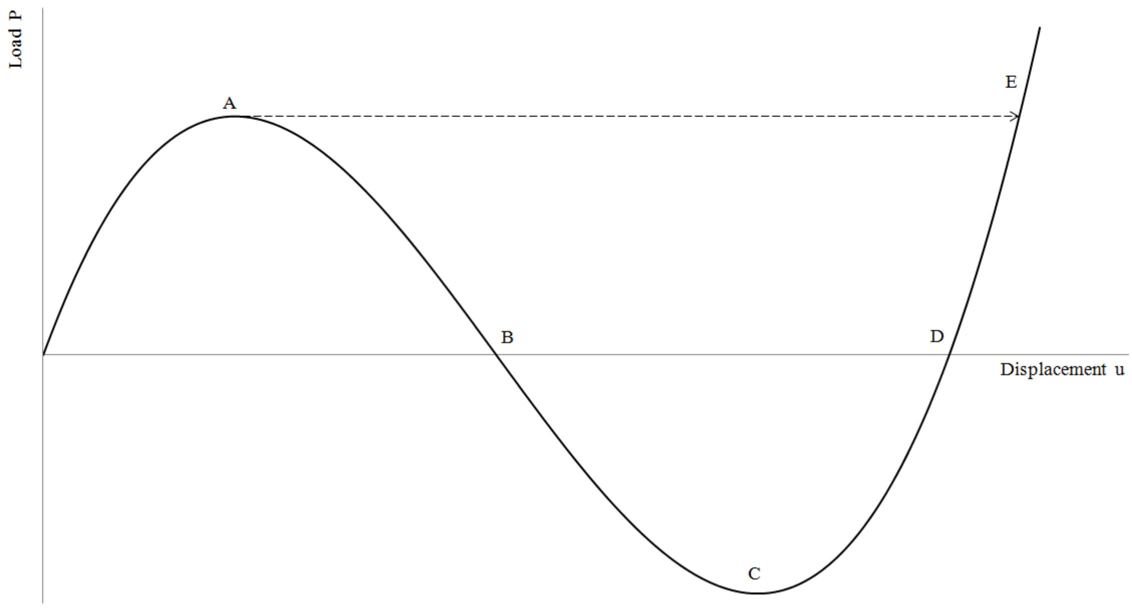

Figure 2: Analytical Load-Displacement Curve

3 Model and Results

The properties of the model are defined in Table 1. In the load-control approach, the load is applied in significantly small increments in order to be able to capture point A. In the displacement-control, the displacement increments are of $1\ mm$. The load-displacement curve for both approaches is presented in Fig. 3 and compared to the analytical solution. If we solve Eq. 1 with respect to the limit points, we observe that at point A the displacement is $10.57\ mm$ and the corresponding critical loading is $P_{cr} = 9.6225\ N$.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 500 \times 10^3\ N/mm^2$	$H = 25\ mm$	$P = 10\ N$
$\nu = 0.4999 \approx 0.5$	$L = 2500\ mm$	$\Delta u = 1\ mm$
	$A = 100\ mm^2$	

Fig. 3 shows that the load-control approach reaches the first limit point and suddenly snaps to the new equilibrium state, corresponding to point E in Fig. 2. The value of the load obtained before the snap-through occurs, corresponding to point A, is $P = 9.62\ N$, with a displacement of $10.415\ mm$, which is in very good agreement with the theoretical critical value P_{cr}. Furthermore, we can observe that the more suitable solution strategy, for such a simple system, for obtaining the load-deflection response is to adopt the displacement control approach, which clearly as shown in Fig. 3, has no difficulty with the local limit point at A and traces the complete equilibrium path.

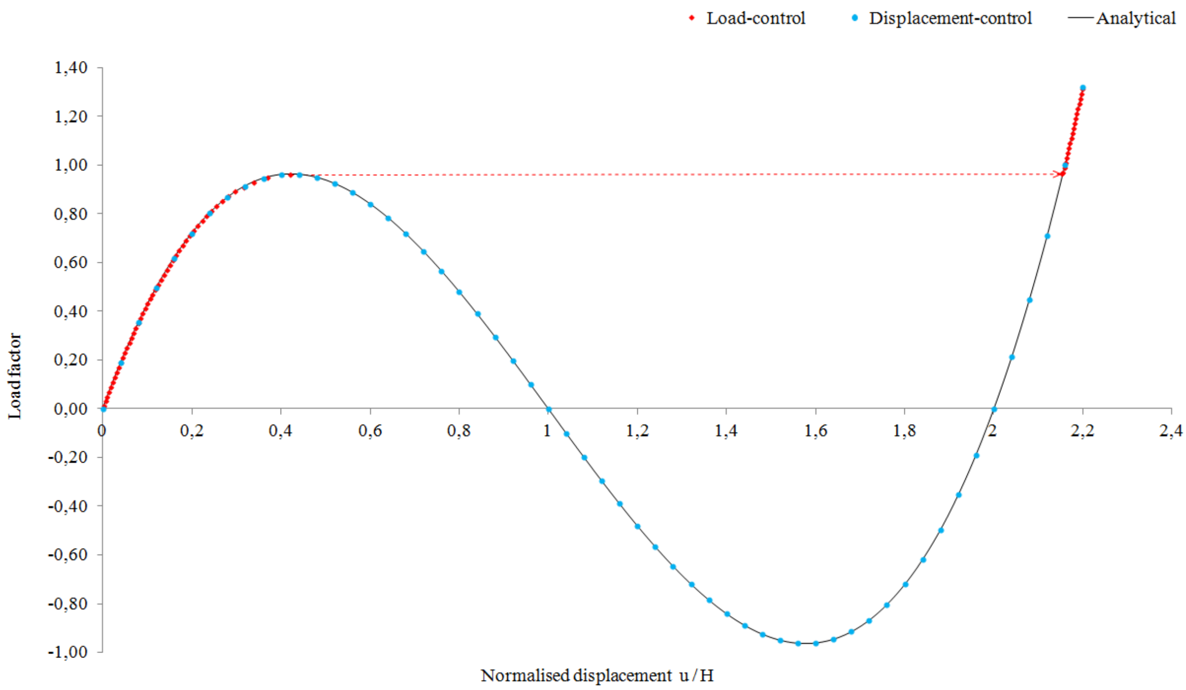

Figure 3: Calculated Load-Displacement Curve

4 Conclusion

This example verifies the determination of the limit load and snap-through behaviour of a simple truss. It has been shown that the geometric non-linear behaviour of the model is captured accurately.

5 Literature

[1] M. A. Crisfield. *Non-linear Finite Element Analysis of Solids and Structures - Volume 1*. John Wiley, 1991.

[2] A. A. Becker. *Background to Finite Element Analysis of Geometric Non-linearity Benchmarks*. Tech. rep. NAFEMS, 1998.