Benchmark Example No. 4

Tie Rod with Lateral Loading

SOFiSTiK | 2018

VERiFiCATiON MANUAL
BE4: Tie Rod with Lateral Loading

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Front Cover

Project: New SOFiSTiK Office, Nuremberg | Contractor: WOLFF & MLLER, Stuttgart | Architecture: WABE-PLAN ARCHITEKTUR, Stuttgart | Structural Engineer: Boll und Partner. Beratende Ingenieure VBI, Stuttgart | MEP: GM Planen + Beraten, Griesheim | Lead Architect: Gerhard P. Wirth gpwirtharchitekten, Nuremberg | Vizualisation: Armin Dariz, BiMOTiON GmbH

Overview	
Element Type(s):	B3D
Analysis Type(s):	STAT, GNL
Procedure(s):	
Topic(s):	
Module(s):	ASE
Input file(s):	tie_rod.dat

1 Problem Description

A tie rod is subjected to the action of a tensile force N and a lateral load P applied at the middle as shown in Fig. (1). Determine the maximum deflection δ_{max}, the slope θ at the left-hand end and the maximum bending moment M_{max}. In addition, compare these three quantities for the case of the unstiffened tie rod ($N = 0$).

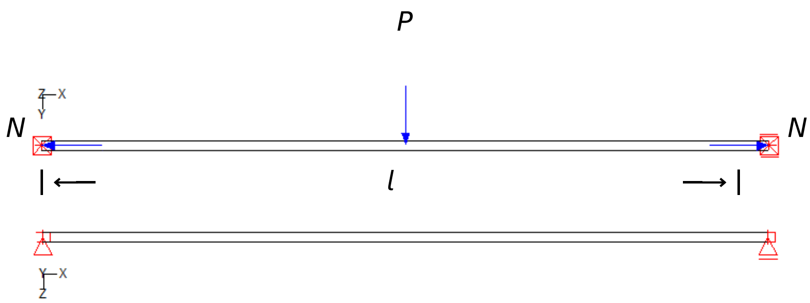

Figure 1: Tie Rod

2 Reference Solution

The combination of direct axial force and lateral load applied at a beam influences the reaction of the structure. Assuming that the lateral force acts in one of the principal planes of the beam and that the axial force is centrally applied by two equal and opposite forces, the expressions for the deflections can be derived from the differential equations of the deflection curve of the beam [1]. Under tension, the maximum deflections of a laterally loaded beam decrease whereas under compression they increase. The moments of the structure are influenced accordingly.

For the simple problem of a beam with hinged ends, loaded by a single force P at the middle, the maximum deflection at the middle is:

$$\delta_{max} = \frac{Pl^3}{48EI}, \tag{1}$$

where l is the lenght of the beam and EI its flexural rigidity. The slope θ at both ends is:

$$\theta = \pm \frac{Pl^2}{16EI}. \tag{2}$$

The maximum value of the bending moment at the middle is:

$$M_{max} = \frac{Pl}{4}. \tag{3}$$

When now the structure (Fig. 1) is submitted to the action of tensile forces N in addition to the initial lateral load P, the deflection at the middle becomes [1]:

$$\delta_{max} = \frac{Pl^3}{48EI} \cdot \frac{u - \tanh u}{\frac{1}{3}u^3}, \tag{4}$$

where $u^2 = Nl^2/4EI$. The first factor in Eq. (4) represents the deflection produced by the lateral load P acting alone. The second factor indicates in what proportion the deflection produced by P is magnified by the axial tensile force N, respectively. When N is small, it approaches unity, which indicates that under this condition the effect on the deflection of the axial force is negligible. The expressions for the moment and the slopes can be derived accordingly [1].

3 Model and Results

The properties of the model are defined in Table 1 and the results are presented in Table 2. Fig. 2 shows the deformed structure under tension and lateral loading.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 30000\, MPa$	$l = 2\, m$	$P = 0.1\, kN$
	$h = 30\, mm$	$N = 0.1\, kN$
	$b = 30\, mm$	
	$I = 6.75 \times 10^{-8}\, m^4$	

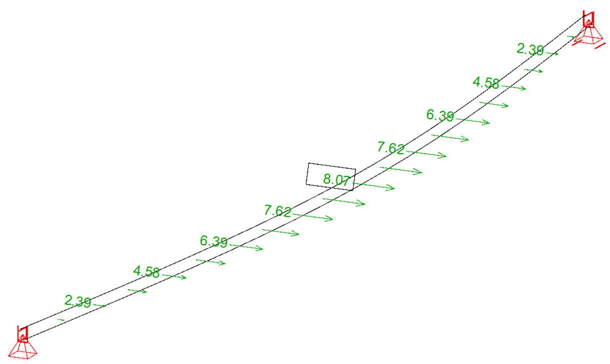

Figure 2: Deformed Structure [mm]: $N \neq 0$

Table 2: Results

	$N = 0$	Ref.	$N \neq 0$	Ref.
$\delta_{max}\,[m]$	0.00823	0.00823	0.00807	0.00807
$M_{max}\,[kNm]$	0.05000	0.05000	0.04919	0.04919
$\theta\,[rad]$	0.01235	0.01235	0.01210	0.01210

4 Conclusion

This example presents the influence of axial forces applied at a laterally loaded beam. The case of a tie rod is examined and the maximum deflections and moment are derived. It has been shown that the behaviour of a beam under the combination of direct axial force and lateral load can be adequately captured.

5 Literature

[1] S. Timoshenko. *Strength of Materials, Part II, Advanced Theory and Problems*. 2nd. D. Van Nostrand Co., Inc., 1940.