Benchmark Example No. 5

Bending of a T-beam
VERIFICATION MANUAL
BE5: Bending of a T-beam

VERIFICATION MANUAL, Version 2018-15
Software Version: SOFISTIK 2018

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The manual and the program have been thoroughly checked for errors. However, SOFISTIK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.

Front Cover
Overview

Element Type(s): B3D
Analysis Type(s): STAT
Procedure(s): 
Topic(s): 
Module(s): AQB, ASE
Input file(s): t_beam.dat

1 Problem Description

An asymmetric T-beam is supported as shown in Fig. 1 and subjected to uniform bending $M_z$. Determine the maximum tensile and compressive bending stresses.

![Figure 1: Model Properties](image)

2 Reference Solution

According to the discussion in Benchmark Example no. 3, it follows that the maximum tensile and compressive stresses in a beam in pure bending are proportional to the distances of the most remote fibers from the neutral axis of the cross-section. When the centroid of the cross-section is not at the middle of the depth, as, for instance, in the case of a T-beam, let $h_1$ and $h_2$ denote the distances from the neutral axis to the outermost fibers in the downward and upward directions (Fig. 1) respectively. Then for a bending moment $M_z$, we obtain the maximum tensile and compressive stresses [1]:

$$
\sigma_{\text{max}} = \frac{M_z h_1}{I_z} \quad \text{and} \quad \sigma_{\text{min}} = -\frac{M_z h_2}{I_z}.
$$

3 Model and Results

The properties of the model are defined in Table 1. Distances from the centroid to the top and bottom of the beam are calculated as 14 cm and 6 cm respectively. The results are presented in Table 2. Figure 2 shows the distribution of the stresses along the cross-section.
Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = 30000 \text{ MPa}$</td>
<td>$l = 1 \text{ m}$</td>
<td>$M_z = 100 \text{ kNm}$</td>
</tr>
<tr>
<td></td>
<td>$h = 20 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_1 = 6 \text{ cm}, h_2 = 14 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b = 9 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_{\text{web}} = 1.5 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_{\text{flange}} = 4 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_z = 2000 \text{ cm}^4$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{max}} \text{ [MPa]}$</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>$\sigma_{\text{min}} \text{ [MPa]}$</td>
<td>$-700$</td>
<td>$-700$</td>
</tr>
</tbody>
</table>

Figure 2: Distribution of Stresses

4 Conclusion

This example shows the derivation of stresses for beams with asymmetric cross-section in which the centroid of the cross-section is not at the middle of the depth. It has been shown that the behaviour of the beam is captured with an excellent accuracy.

5 Literature