Benchmark Example No. 52

Verification of Wave Kinematics

SOFiSTiK | 2018
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The manual and the program have been thoroughly checked for errors. However, SOFiSTiK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
1 Problem Description

This benchmark is concerned with the validation of wave kinematics of regular nonlinear Stokes 5\textsuperscript{th} order wave theory. In Fig. 1 the properties of a wave can be visualised.

![Wave Image]

Figure 1: Wave

2 Reference Solution

The reference solution is provided in [1]. This article investigates the solution of the dispersion relation of Stokes fifth order wave theory, which is governed by two coupled nonlinear equations in two variables, through a Newton-Raphson iterative scheme. Different waves are investigated and their wave profile and horizontal velocity is computed and plotted. The interest of this benchmark focuses on the provided solution for the corrected coefficient in the original expression for $C_2$ (the factor $+2592$ should be replaced by $-2592$), which is employed also from SOFiSTiK. For more information on this correction please refer to Nishimura & al. (1977), Fenton (1985) [2], Bhattacharyya (1995) [1] and SOFiLOAD manual [3].

3 Model and Results

The properties of the considered wave are defined in Table 1.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Wave Properties</th>
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<tbody>
<tr>
<td>$d = 107 \text{ ft}$</td>
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<tr>
<td>$H = 70 \text{ ft}$</td>
</tr>
<tr>
<td>$T = 16.30 \text{ s}$</td>
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</tbody>
</table>
The wave profile, i.e. the phase angle $\theta$ versus the surface elevation $\eta$, is computed and shown in Fig 2 and the horizontal velocity under the wave crest versus the elevation from the seabed ($z - d$), in Fig 3. Both results are compared to the reference solution, as presented in Bhattacharyya (1995) [1].
4 Conclusion
The very good agreement between the reference and the results computed by SOFiSTiK verifies that the Stokes fifth order wave theory is adequately implemented.

5 Literature