Benchmark Example No. 55

Design elements and SOFiSTiK T-beam Philosophy

SOFiSTiK | 2018
1 Problem Description
This verification sets a benchmark for Design Elements in SOFiSTiK 2018 at the analysis level. Results are compared with those of the existing SOFiSTiK T-Beam Philosophy, and a reference analytical solution.

2 Reference Solution
As an example, the solution of the bending moment at the middle support $M_{y,s}$ of a two span continuous T-beam system shown in figure 1 is considered. The effective width of the beam is the determining parameter. The exact analytical value of the support moment is given by:

$$M_{y,s} = \frac{w l^2}{8}$$

Figure 1: Reference system: continuous beam and its bending moment distribution

For bending design of a T-beam, analysis can be done in the module ASE using the SOFiSTiK T-beam Philosophy [1, 2, 3, 4]. Accordingly, the resulting bending moment $M_{y,Tbeam}$ is calculated by multiplying the effective width $b_{eff}$ with the nodal value of the plate elements that is computed by averaging some of the nodal values of the plates the at the support (eg. $m_{y,avg} = (m_1 + m_2 + m_3)/3$), then adding
the remaining moment carried by the beam element $M_{y,beam}$. That is:

$$M_{y,Tbeam} = M_{y,beam} + m_{y,avg} \cdot b_{eff}$$  \hspace{1cm} (2)

### 3 Model and Results

The wide flange of the continuous T-beam is modeled with quad elements. In addition, a centric beam with a T-cross section is embedded at the center of the flange. A uniformly distributed area load is then applied over the flange. To supplement the comparison, three variations of this model are made, in which only the effective width is altered.

In contrast to the SOFiSTiK T-Beam Philosophy stated in equation 2, the bending moment calculation by Design Elements takes the integral of all the nodal values covered within the effective width, then adds the remaining beam moment. This improves the approximation, and can be put as:

$$M_{y,Tbeam} = M_{y,beam} + \int_0^{b_{eff}} m_{y,p}(l) \, dl$$ \hspace{1cm} (3)

Figure 3 illustrates this difference.
As a result, applying the T-beam Philosophy on a model with a larger effective width, the error in the support bending moment is more pronounced (see table 1). However, the results of the Design Elements, as one may expect, remain well approximate for all effective widths. In this example, larger widths were so chosen that the deviations could clearly be illustrated.

Table 1: Comparison of support moments for varying effective widths

<table>
<thead>
<tr>
<th>Effective Widths $b_{\text{eff},i} [m]$</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. $[kNm]$</td>
<td>$-32.4$</td>
<td>$-64.8$</td>
<td>$-129.6$</td>
</tr>
<tr>
<td>SOF. T-Beam Phil. $[kNm]$</td>
<td>$-32.2$</td>
<td>$-66.2$</td>
<td>$-148.8$</td>
</tr>
<tr>
<td>$</td>
<td>e</td>
<td>[%]$</td>
<td>0.6</td>
</tr>
<tr>
<td>Design Elements $[kNm]$</td>
<td>$-32.0$</td>
<td>$-64.0$</td>
<td>$-129.3$</td>
</tr>
<tr>
<td>$</td>
<td>e</td>
<td>[%]$</td>
<td>1.2</td>
</tr>
</tbody>
</table>

**4 Conclusion**

In case of a flexural design of a finite element structural model consisting T-beams, the results using the Design Elements and the SOFiSTiK T-beam Philosophy match that of the exact analytical solution up to a certain effective width. The design elements however always ensure more approximate results regardless of the effective width.

**5 Literature**