Benchmark Example No. 7

Large Deflection of Cantilever Beams I

SOFiSTiK | 2018

VERiFiCATiON MANUAL
BE7: Large Deflection of Cantilever Beams I

VERiFiCATiON MANUAL, Version 2018-7
Software Version: SOFiSTiK 2018

SOFiSTiK AG

HQ Oberschleissheim
Bruckmannring 38
85764 Oberschleissheim
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

Office Nuremberg
Burgschmietstr. 40
90419 Nuremberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

info@sofistik.com
www.sofistik.com

Front Cover

Project: New SOFiSTiK Office, Nuremberg | Contractor: WOLFF & MLLER, Stuttgart | Architecture: WABE-PLAN ARCHITEKTUR, Stuttgart | Structural Engineer: Boll und Partner. Beratende Ingenieure VBI, Stuttgart | MEP: GM Planen + Beraten, Griesheim | Lead Architect: Gerhard P. Wirth gpwirtharchitekten, Nuremberg | Vizualisation: Armin Dariz, BiMOTiON GmbH

Overview

Element Type(s):	B3D, SH3D
Analysis Type(s):	STAT, GNL
Procedure(s):	LSTP
Topic(s):	
Module(s):	ASE
Input file(s):	beam_elem.dat, quad_elem.dat

1 Problem Description

A cantilever beam is supported as shown in Fig. 1. The beam is subjected to a total vertical load, applied at the tip of the cantilever, which should cause the tip to deflect significantly. The determination of the non-dimensional tip deflections ratios are determined.

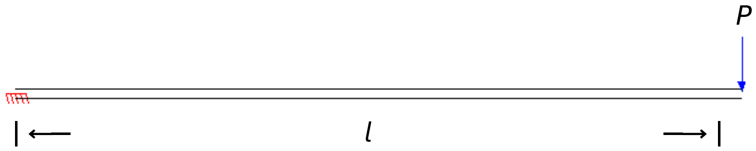

Figure 1: Model Properties

2 Reference Solution

The classical problem of deflection of a cantilever beam of linear elastic material, under the action of an external vertical concentrated load at the free end, is analysed (Fig. 2). The solution for large deflection of a cantilever beam cannot be obtained from elementary beam theory since basic assumptions are no longer valid. The elementary theory includes specific simplifications e.g. in the consideration of curvature derivatives, and provides no correction for the shortening of the moment arm as the loaded end of the beam deflects. For large finite loads, it gives deflections greater than the length of the beam [1].

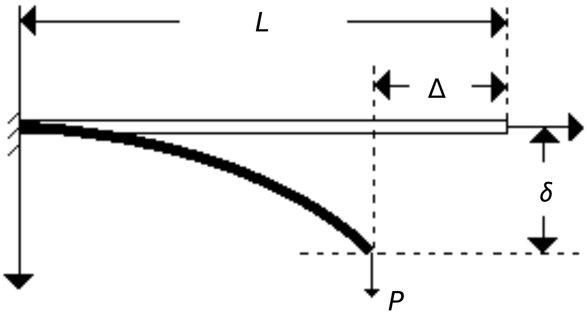

Figure 2: Problem Definition

The mathematical treatment of the equilibrium of cantilever beams does not involve great difficulty. Nevertheless, unless small deflections are considered, an analytical solution does not exist, since for large deflections a differential equation with nonlinear terms must be solved. The problem is said to involve geometrical nonlinearity [2]. Therefore in order to account for this nonlinear term, third order theory is performed, where the equilibrium is established at the deformed configuration (geometrically nonlinear analysis).

3 Model and Results

A circular pipe with cross-section of outer diameter $0.2\ m$ and wall thickness $0.01\ m$ is used, so that the beam is moderately slender. This type of problem becomes considerably more difficult numerically as the slenderness ratio increases [3]. The finite element model consists of twenty elements. The properties of the model are defined in Table 1.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 100\ MPa$	$l = 10\ m$	$P = 269.35\ N$
	$D = 0.2\ m$	
	$t = 0.01\ m$	

As an alternative, the structure is analysed with quad plane elements with a cross-section of the same stiffness as the circular, in order to achieve the same results and compare the behaviour of the two types of elements. The quad cross-section has a width of $0.3\ m$ and a thickness of $0.10261\ m$, and therefore the same moment of inertia $I = 2.701\ m^{-5}$ as the one of the circular cross-section. Results for both models are presented in Table 2. Figure 3 shows the deflection of the beam for the two analysed models. Figure 4 presents the results, in terms of the motion of the tip of the cantilever, where they are compared to the exact solution for the inextensible beam, as given by Bisshopp and Drucker [1].

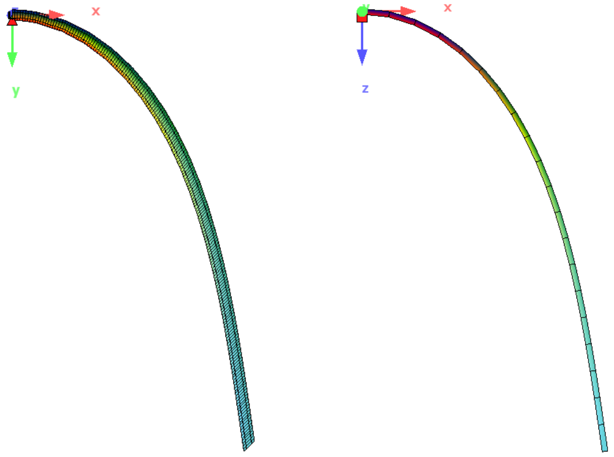

Figure 3: Deformed structure: a) Beam elements b) Quad elements

Table 2: Results

	Beam	Quad
$\delta[m]$	8.113	8.102
$\Delta[m]$	5.545	5.539

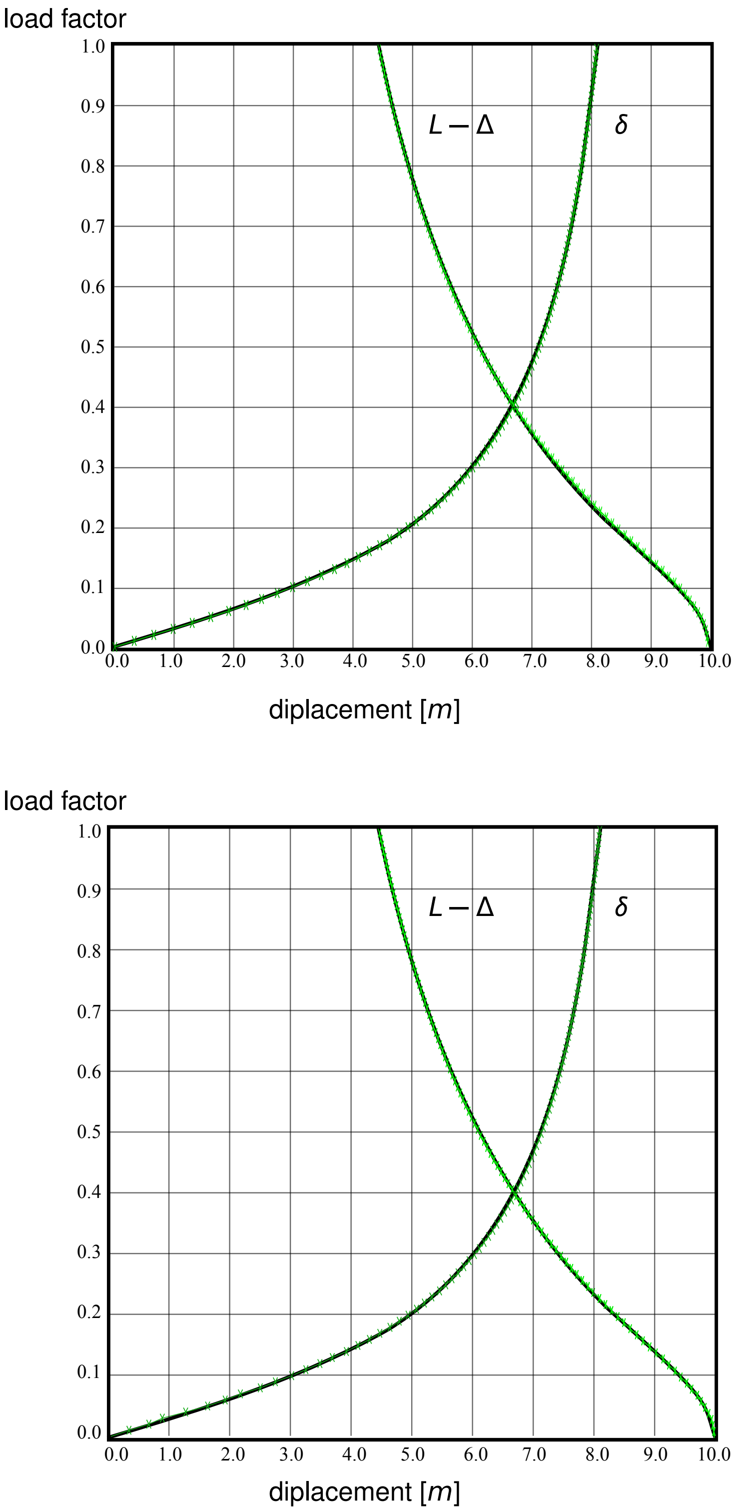

Figure 4: Load - Deflection: (a). Beam elements (b). Quad elements

4 Conclusion

This benchmark shows the classical problem of a cantilever beam undergoing large deformations under the action of a vertical load at the tip. Results are presented in terms of the motion of the tip of the cantilever where the accuracy of the solution is apparent.

5 Literature

[1] K. E. Bisshopp and D. C. Drucker. “Large Deflection of Cantilever Beams”. In: *Quarterly of Applied Mathematics* 3 (1945), pp. 272–275.

[2] T. Beléndez, C. Neipp, and A. Beléndez. “Large and Small Deflections of a Cantilever Beam”. In: *European Journal of Physics* 23.3 (2002), pp. 371–379.

[3] *Abaqus Benchmarks Manual 6.10*. Dassault Systmes Simulia Corp. 2010.