Benchmark Example No. 9

Verification of Beam and Section Types I

SOFiSTiK | 2018

VERiFiCATiON MANUAL
BE9: Verification of Beam and Section Types I

VERiFiCATiON MANUAL, Version 2018-7
Software Version: SOFiSTiK 2018

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Front Cover

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Overview	
Element Type(s):	B3D
Analysis Type(s):	STAT
Procedure(s):	
Topic(s):	
Module(s):	AQUA
Input file(s):	cross_sections_FEM.dat,cross_sections_BEM.dat

1 Problem Description

In this Benchmark different cross-section types are investigated, in order to test the properties of each cross-section associated with their definition in AQUA module. The analysed non-tabulated and tabulated cross sections are shown in Figures 1 and 2, respectively.

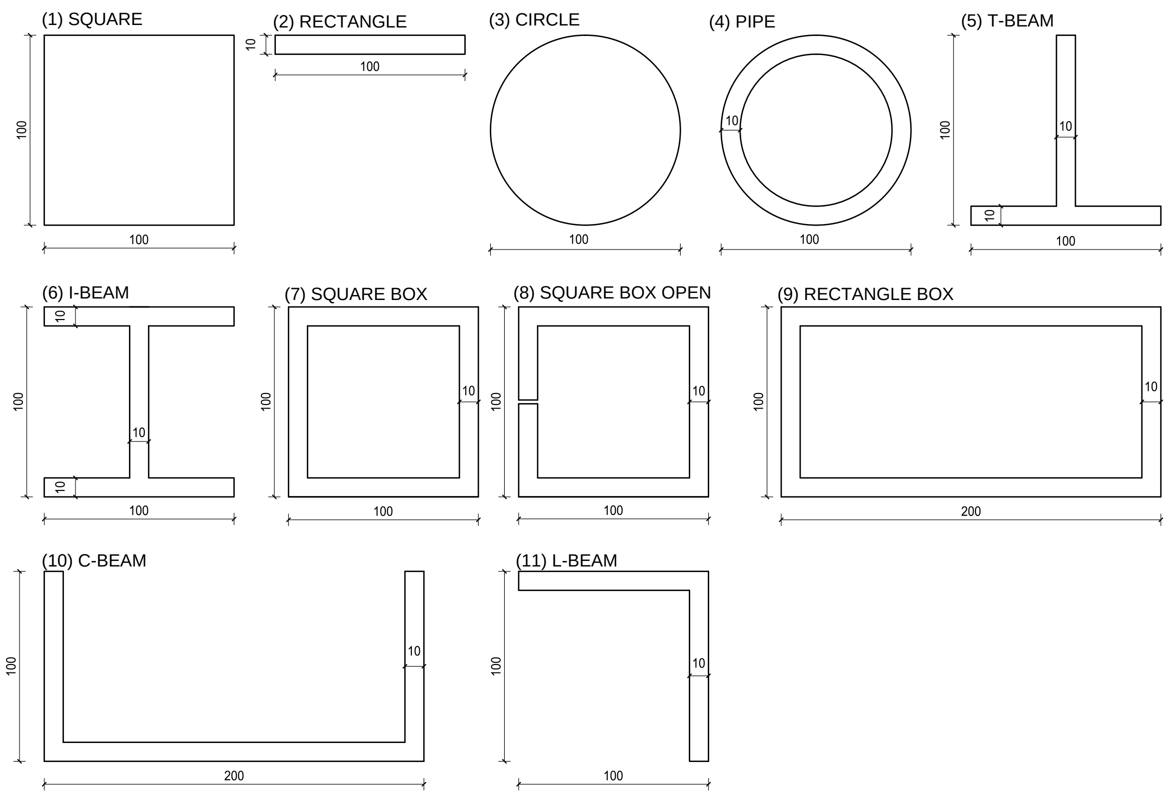

Figure 1: Non-tabulated Cross Sections

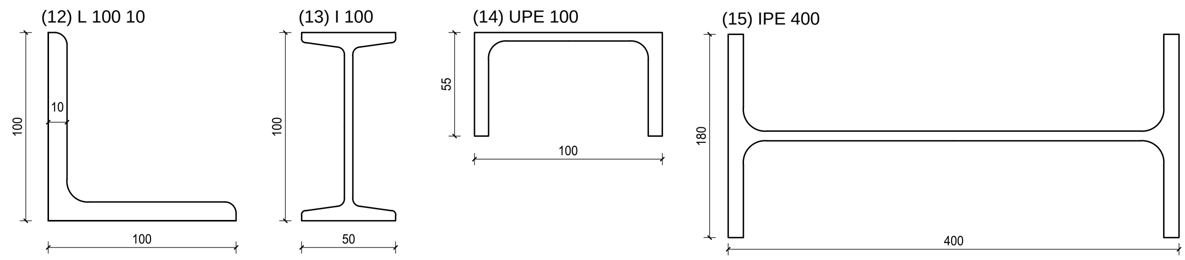

Figure 2: Tabulated Cross Sections

2 Reference Solution

The important values of a cross-section for the simple cases of bending and torsion are the moment of inertia and the torsional moment, respectively. The analytical solution for the moment of inertia I_y with respect to y axis is [1]:

$$I_y = \int_A z^2 dA, \tag{1}$$

in which each element of area dA is multiplied by the square of its distance from the z-axis and the integration is extended over the cross-sectional area A of the beam (Fig. 3). The torsional moment I_T is more complicated to compute and depends on the cross-sections geometry. For circular cross-sections is:

$$I_T = \int_A r^2 dA, \tag{2}$$

For thick-walled non-circular cross-sections, it depends on the warping function. Tabulated formulas are given in all relevant handbooks for the most common geometries [2]. For closed thin-walled non-circular cross-sections I_T is [3]:

$$I_T = \frac{4A_m^2}{\sum_{i=1}^{n} \frac{s_i}{t_i}}, \tag{3}$$

and for open thin-walled non-circular cross-sections is:

$$I_T = \frac{1}{3}\sum_{i=1}^{n} s_i t_i^3, \tag{4}$$

where A_m is the area enclosed from the center line of the wall (Fig. 3), and t_i, s_i the dimensions of the parts from which the cross-section consists of. For the specific case of an I-cross-section, another approximate formula can be utilised, as defined by Petersen [3]:

$$I_T = 2\frac{1}{3}\, b\, t^3 \left(1 - 0.630\,\frac{t}{b}\right) + \frac{1}{3}\,(h-2t)\; s^3 + 2\,\alpha\, D^4, \tag{5}$$

where s, t and D are described in Fig. 3 and α is extracted from the corresponding diagram, given in [3], w.r.t. the cross-section properties. For the same cross-section but according to Gensichen, I_T is accordingly computed as:

$$I_T = 2\frac{1}{3}\,b\,t^3\left(1-0.630\,\frac{t}{b}\right)+\frac{1}{3}\,(h-2t)\,s^3+0.29\,\frac{s}{t}\left[\frac{\left(\frac{s}{2}\right)^2+t^2}{t}\right]^4 \tag{6}$$

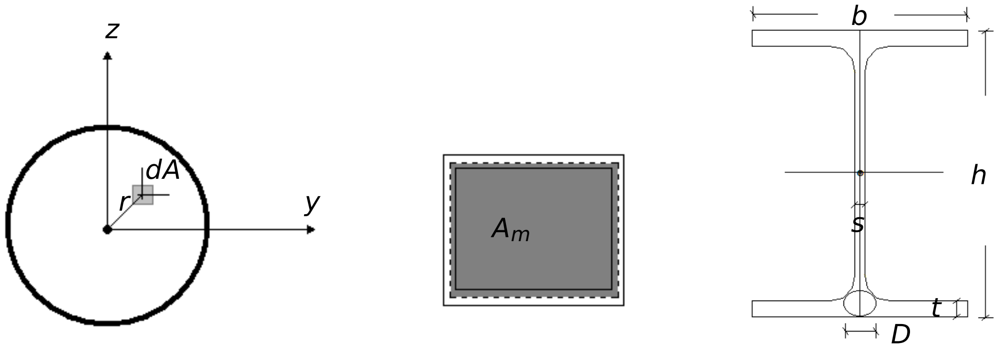

Figure 3: Cross-Sectional Properties

3 Model and Results

The properties of different cross-sections, analysed in this example, are defined in Table 1. The cross-sections types are modelled in various ways in AQUA and compared. For differentiation between them, the modelling type is specified next to the name of each cross-section. The cross-sectional properties of the thick walled sections are computed by implementing the finite element method (FEM).

Table 1: Cross-Sections Properties

Material Properties	Cross-sectional Properties
$E = 30\,MPa$	$b = 100\,mm$
$\nu = 0.3$	$h = 100\,mm$
	$t = 10\,mm$
	$D = 100\,mm$

Table 2: Results

	$I_y\ [cm^4]$		$\lvert e_r \rvert$	$I_T\ [cm^4]$		$\lvert e_r \rvert$
Type	SOF.	Ref.	[%]	SOF.	Ref.	[%]
(1) Square -srec	833.33	833.33	0.00	1405.78	1400.00 (4)	0.41
(2) Rectangle -srec	0.83	0.83	0.00	3.12	3.13 (4)	0.22
(3) Circle -scit	490.87	490.87	0.00	981.75	981.75 (4)	0.00
(4) Pipe -scit	289.81	289.81	0.00	579.62	579.62 (4)	0.00

Table 2: (continued)

	I_y [cm^4]		$\|e_r\|$	I_T [cm^4]		$\|e_r\|$
Type	SOF.	Ref.	[%]	SOF.	Ref.	[%]
(5) T-beam -poly	180.00	180.00	0.00	6.37	6.33 (4)	0.63
(5) T-beam -plat	181.37	182.82	0.79	6.50	6.50 (4)	0.00
(6) I-beam -poly	449.33	449.33	0.00	9.67	9.33 (4)	3.64
					9.62 (6)	0.52
					9.21 (5)	4.99
(6) I-beam -plat	465.75	467.42	0.36	9.67	9.33 (4)	3.64
(6) I-beam -weld	447.67	449.33	0.37	9.33	9.33 (4)	0.00
(7) Square box -poly	492.00	492.00	0.00	796.78	729.00 (4)	9.30
				772.34[1]	729.00 (4)	5.95
(7) Square box -plat	486.00	487.50	0.31	741.00	729.00 (4)	1.65
(8) Square box open -plat	486.00	487.50	0.31	11.98	12.00 (4)	0.17
(9) Rectang. box -poly	898.67	898.67	0.00	2221.56	2088.64 (4)	6.36
				2168.44[1]	2088.64 (4)	3.82
(9) Rectang. box -plat	891.00	889.17	0.21	2107.31	2088.64 (4)	0.89
(10) C-beam -poly	2292.67	2292.67	0.00	12.76	12.67 (4)	0.73
(10) C-beam -plat	2286.33	2287.92	0.07	12.67	12.67 (4)	0.0
(11) L-beam -poly	180.00	180.00	0.00	6.26	6.33 (4)	1.22
(11) L-beam -weld	179.25	180.00	0.42	6.33	6.33 (4)	0.0
(11) L-beam -plat	178.62	179.40	0.44	6.33	6.33 (4)	0.0
(12) L 100 10 (tabulated)	176.68	177.0 [4]	0.18	6.83	6.33 [5]	7.88
(13) I 100 (tabulated)	170.21	171.0 [4]	0.46	1.51	1.60 [4]	5.68
		170.3 [5]	0.06		1.511 [5]	0.12
(14) UPE 100 (tabulated)	206.85	207.0 [4]	0.07	2.02	1.99 [4]	1.45
		206.9 [5]	0.03		2.01 [5]	0.44
(15) IPE 400 (tabulated)	23126.97	23130 [4]	0.01	50.32	51.40 [4]	2.09
		23128 [5]	0.00		50.41 [5]	0.17

1 Calculated with a finer mesh: HDIV 2[mm]

From the results in Table 2 we can see that for the definition of general cross-sections the use of -POLY option gives the exact values for I_y. When evaluating the results of the torsional moment of inertia I_T, it has to be taken into consideration, that the presented reference solutions in Sect. 2, for all non-circular cross-sections, are approximate and various assumptions are taken according to the adopted theory. For the case of the I-beam, it is observed in Table 2, that the relative error ranges between 4.99 % and 0.52 %.

For the definition of thin-walled cross-sections the use of -PLAT gives very good results for I_T whereas for the determination of I_y some deviations appear. This is due to the fact that in order for the cross-section to be connected for shear, some parts of the plates overlap at the connections giving an additional moment of inertia around the y-axis. This can be seen at Fig. 4 for the I beam. It can be avoided if the -PLAT option is used without overlapping of parts but in combination with -WELD in order to ensure the proper connection of the plates. This can be seen from the results for the I- and L-beam which are analysed for the three options -POLY, -PLAT, -PLAT and -WELD.

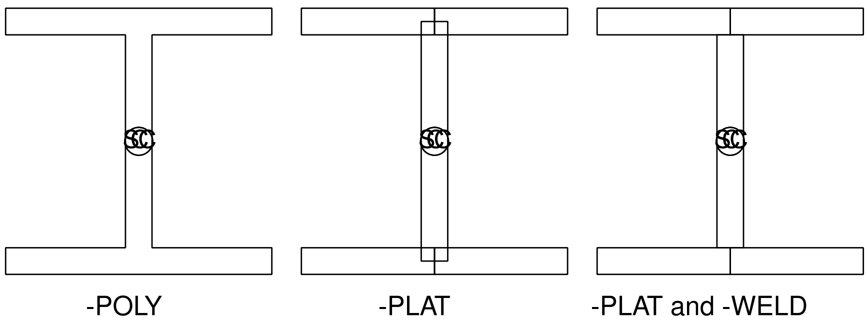

Figure 4: Definition types of I-beam

3.1 Comparison of numerical approaches for thick walled cross sections

The torsional moment of inertia is additionally calculated for the thick walled non-circular cross sections by using the boundary element method (BEM). The computed values are compared with the results obtained from the FEM method and with the reference values in Table 3.

Table 3: Torsional moment of inertia calculated by using the boundary element method (BEM) and the finite element method (FEM)

	I_t $[cm^4]$			$\lvert e_r \rvert$ BEM/FEM
Type	SOF. BEM	SOF. FEM	Ref.	[%]
(5) T-beam -poly	6.45	6.37	6.33 (4)	1.89/0.63
(6) I-beam -poly	9.52	9.67	9.33 (4)	2.00/3.64
			9.62 (6)	1.04/0.52
			9.21 (5)	3.36/4.99
(7) Square box -poly	771.96	772.34[1]	729.00 (4)	5.89/5.94
(9) Rectang. box -poly	2171.77	2168.44[1]	2088.64 (4)	3.98/3.82
(10) C-beam -poly	13.29	12.76	12.67 (4)	4.90/0.73
(11) L-beam -poly	6.36	6.26	6.33 (4)	0.36/1.22
(12) L 100 10 (tabulated)	6.93	6.83	6.33 [5]	9.41/7.88

Table 3: (continued)

Type	I_t [cm^4] SOF. BEM	SOF. FEM	Ref.	$\lvert e_r \rvert$ BEM/FEM [%]
(13) I 100 (tabulated)	1.51	1.51	1.60 [4]	5.44/5.68
			1.511 [5]	0.13/0.12
(14) UPE 100 (tabulated)	2.04	2.02	1.99 [4]	2.37/1.45
			2.01 [5]	1.35/0.44
(15) IPE 400 (tabulated)	50.30	50.32	51.40 [4]	2.15/2.09
			50.41 [5]	0.23/0.17

1 Calculated with a finer mesh: HDIV 2[mm]

3.2 Convergence of the thick walled sections (FEM-BEM) in regard to the thin-walled theory

The reference values for the open sections I, L, C, T-beam are computed with respect to the thin-walled theory reference solution (Eq. 4). Therefore for the calculated values with -POLY (FEM and BEM), which do not correspond to the thin-walled theory, deviations appear. If we now make a convergence study, for the case of the I-beam, decreasing the thickness of the cross-section and comparing it to the thin-walled reference solution, we will observe that the deviation is vanishing as we approach even thinner members. This is presented in Fig. 5 for an I-beam, where the absolute difference of the calculated from the reference value is depicted for the decreasing thickness values. The results obtained with -POLY (FEM) are presented in Fig. 5 with three different mesh sizes: default mesh, 25% and 50%finer mesh.

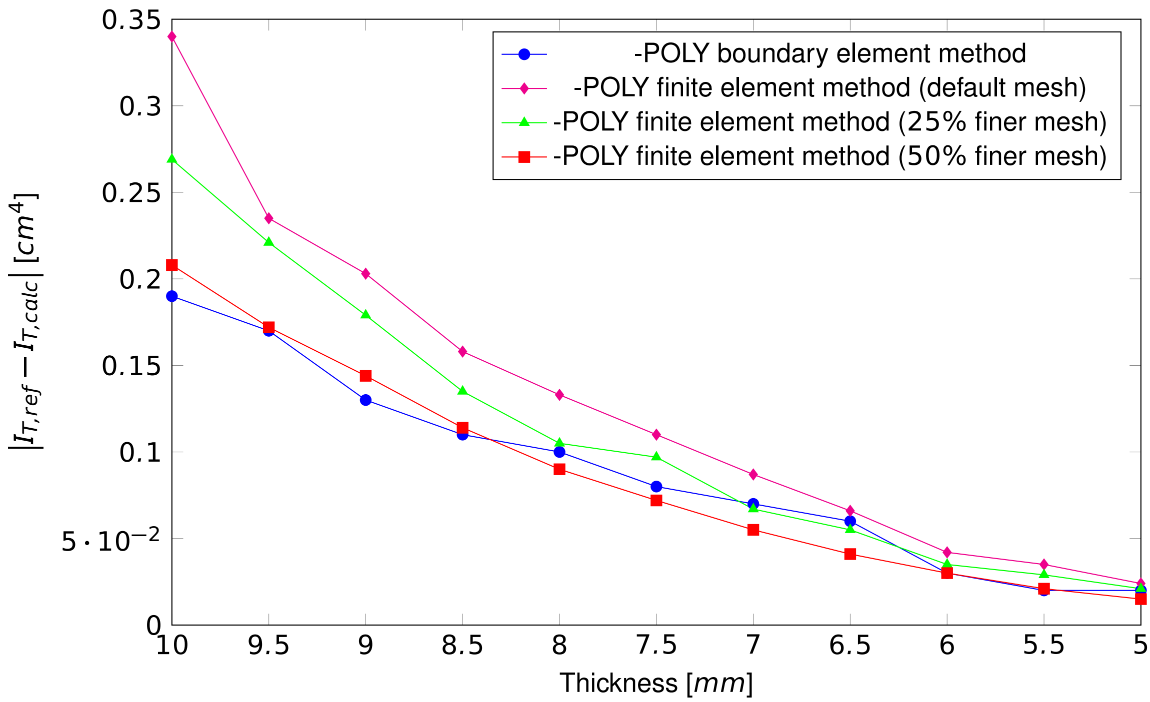

Figure 5: Convergence of I-beam

For the case of open thin-walled non-circular cross-sections, modelled with -PLAT, we can observe that I_T matches exactly the reference solution. For closed thin-walled non-circular cross-sections though, some deviations arise. If we take a closer look at the case of the square box, at first glance it appears to be not accurate enough, since the calculated value is 741.00 cm^4 and the reference is 729.00 cm^4 (Table 2). The difference between them is 741.00 - 729.00 = 12 cm^4, which corresponds to the reference value of the open square box. This is due to the fact, that the reference solution for this type of sections given by Eq. 3, corresponds to the thin-walled theory and assumes a constant distribution of shear stresses over the thickness of the cross-section. However, SOFiSTiK assumes a generalised thin-walled theory, where the shear stresses due to torsion, are distributed linearly across the thickness, as shown in Fig. 6, and thus holds:

$$I_{T_{generalised\,thin-walled\,theory}} = I_{T_{closed,SOFiSTiK}} = I_{T_{closed,thin-walled\,theory}} + I_{T_{open,thin-walled\,theory}} \quad (7)$$

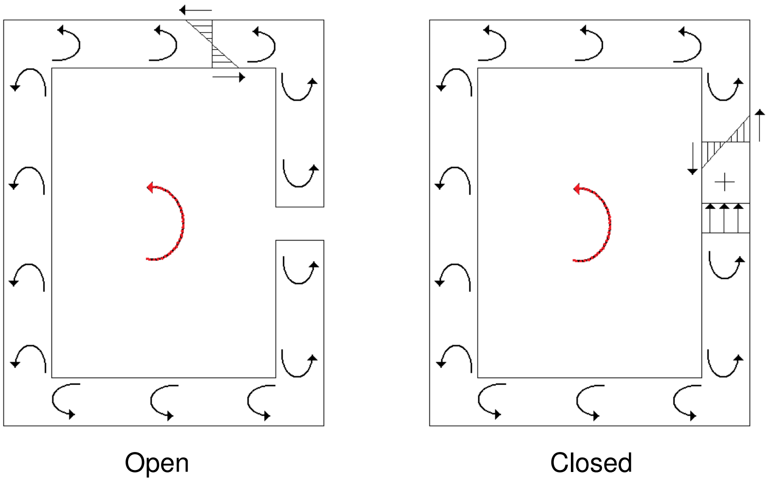

Figure 6: Distribution of Stresses

Eq. 7 is satisfied exactly for the square box cross-section and it can be visualised in Fig. 7 by the purple line for decreasing thicknesses, whereas the blue line denotes the deviation of the calculated values with respect to the $I_{T_{closed,thin-walled\,theory}}$.

For the same cross-section, but now modelled with -POLY, it is evident that the difference from the reference solution is larger, reaching the value of 5.89 %, as presented by the green line. This is due to the fact that except from the difference in the stresses consideration, as explained above, the thin-walled assumption is also engaged. If we do a convergence study for this cross-section, and compared it to the one modelled with -PLAT, represented by the red line, we will observe that as the thickness decreases the deviation curves gradually coincide.

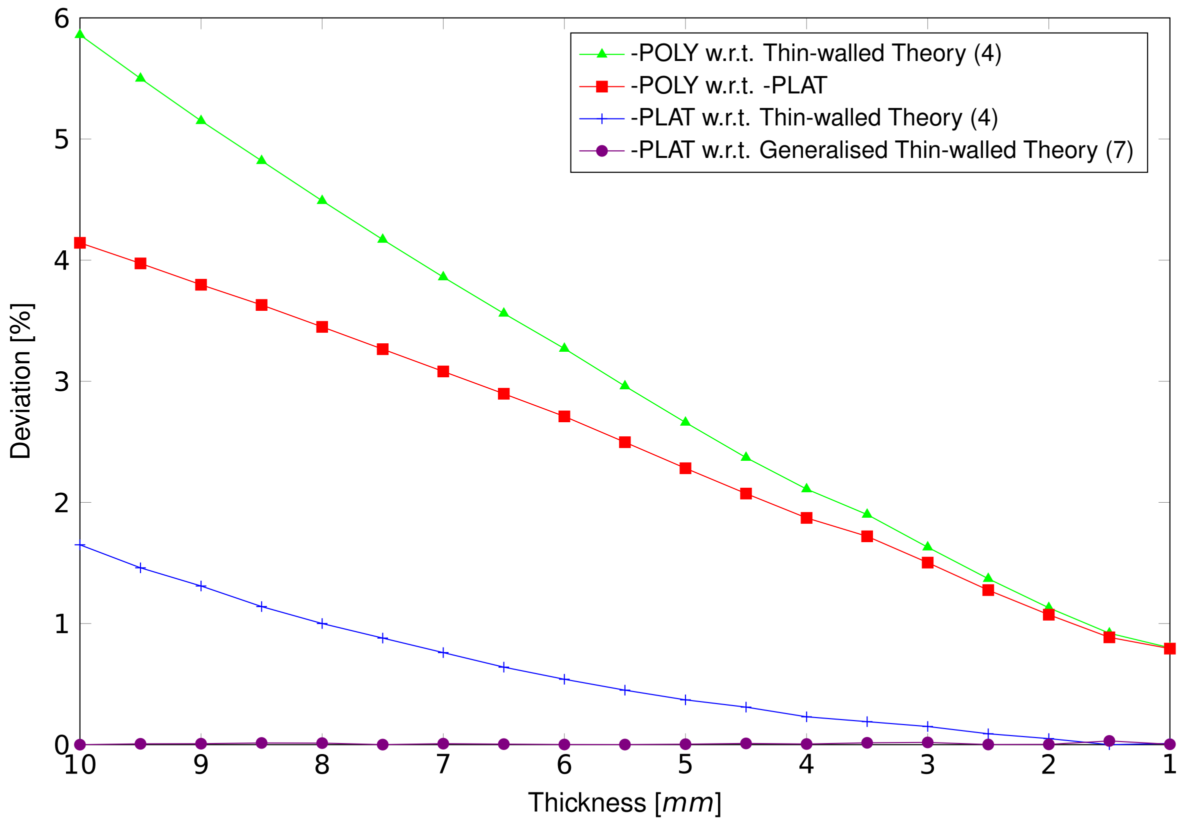

Figure 7: Convergence of Square Box

4 Conclusion

This example presents the different cross-sections and their properties according to their definition in AQUA. It has been shown that the properties of the cross-sections can be adequately captured irrelevantly of their definition with small deviations from the exact solution.

5 Literature

[1] S. Timoshenko. *Strength of Materials, Part I, Elementary Theory and Problems*. 2nd. D. Van Nostrand Co., Inc., 1940.
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