Benchmark Example No. 1

Design of Slab for Bending

SOFiSTiK | 2018
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The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
1 Problem Description

The problem consists of a slab section of depth \( h \), as shown in Fig. 1. The cross-section is designed for an ultimate moment \( m_{Ed} \) and the required reinforcement is determined.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to pure flexure, such as beams or slabs. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Basic assumptions for section design (Section 6.1)
- Reinforcement (Section 9.3.1.1, 9.2.1.1)

![Figure 2: Stress and Strain Distributions in the Design of Cross-sections](image)

In singly reinforced beams and slabs, the conditions in the cross-section at the ultimate limit state, are assumed to be as shown in Fig. 2. The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).
3 Model and Results

The rectangular slab section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1] [2], to carry an ultimate moment of 25 kNm. The calculation steps with different design methods [3] [4] [5] are presented below and the results are given in Table 2. Here, it has to be mentioned that these standard methods employed in order to calculate the reinforcement are approximate, and therefore deviations often occur.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 25/30</td>
<td>h = 20.0 cm</td>
<td>$m_{Ed} = 25$ kNm/m</td>
</tr>
<tr>
<td>B 500A</td>
<td>d = 17.0 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b = 1.0 m</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th>$a_{s1} [cm^2/m]$</th>
<th>SOFiSTiK 2018</th>
<th>General Chart [3]</th>
<th>$\omega$—Table [3]</th>
<th>$k_d$—Table [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.334</td>
<td>3.328</td>
<td>3.334</td>
<td>3.333</td>
</tr>
</tbody>
</table>
4 Design Process\(^1\)

Design with respect to DIN EN 1992-1-1:2004 (NA) \([1]\) \([2]\):\(^2\)

Material:

Concrete: \(\gamma_c = 1.50\)
Steel: \(\gamma_s = 1.15\)

\(f_{ck} = 25\ \text{MPa}\)
\(f_{cd} = a_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 25/1.5 = 14.17\ \text{MPa}\)

\(f_{yk} = 500\ \text{MPa}\)
\(f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ \text{MPa}\)

Design Load:

\(M_{Ed} = m_{Ed} \cdot b = 25\ \text{kNm};\quad N_{Ed} = 0\)
\(M_{Eds} = M_{Ed} - N_{Ed} \cdot z_s = 25\ \text{kNm}\)

Design with respect to General Design Chart Bending with axial force for rectangular cross-sections:

\[
\mu_{Eds} = \frac{M_{Eds}}{b \cdot d^2 \cdot f_{cd}} = \frac{25 \cdot 10^{-3}}{1.0 \cdot 0.17^2 \cdot 14.17} = 0.061
\]

\(\varepsilon = 25 \cdot 10^{-3};\quad \zeta = 0.97\quad \Rightarrow\quad \sigma_{s1d} = 456.52\ \text{MPa}\)

\(a_{s1} = \frac{1}{\sigma_{s1d}} \cdot (\frac{M_{Eds}}{\zeta \cdot d} + N_{Ed}) = 3.328\ \text{cm}^2/\text{m}\)

Design with respect to \(\omega\) (or \(\mu_s\)) Design Table for rectangular cross-sections:

\[
\mu_{Eds} = \frac{M_{Eds}}{b \cdot d^2 \cdot f_{cd}} = \frac{25 \cdot 10^{-3}}{1.0 \cdot 0.17^2 \cdot 14.17} = 0.061
\]

\(\omega = 0.0632\) (interpolated) and \(\sigma_{sd} = 456.52\ \text{MPa}\)

\(a_{s1} = \frac{1}{\sigma_{sd}} \cdot (\omega \cdot b \cdot d \cdot f_{cd} + N_{Ed}) = 3.334\ \text{cm}^2/\text{m}\)

Design with respect to \(k_d\) Design Table for rectangular cross-sections:

\[
k_d = \frac{d}{\sqrt{M_{Eds}/b}} = \frac{17}{\sqrt{25/1.0}} = 3.40
\]

\(k_s = 2.381,\ \kappa_s = 0.952\) (interpolated values)

\(a_{s1} = \left(\frac{k_s}{d} \cdot \frac{M_{Eds}}{N_{Ed}} + \frac{N_{Ed}}{\sigma_{s1d}}\right) \cdot \kappa_s = 3.333\ \text{cm}^2/\text{m}\)

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\(^1\)The tools used in the design process are based on steel stress-strain diagrams, as defined in \([1]\)3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

\(^2\)The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) \([1]\), \([2]\), unless otherwise specified.
5 Conclusion

This example shows the calculation of the required reinforcement for a slab section under bending. Various different reference solutions are employed in order to compare the SOFiSTiK results to. It has been shown that the results are reproduced with excellent accuracy.

6 Literature


