Benchmark Example No. 10

Shear between web and flanges of T-sections
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Overview

<table>
<thead>
<tr>
<th>Design Code Family(s):</th>
<th>DIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Code(s):</td>
<td>DIN EN 1992-1-1</td>
</tr>
<tr>
<td>Module(s):</td>
<td>AQB</td>
</tr>
<tr>
<td>Input file(s):</td>
<td>t-beam_shear_web_flange.dat</td>
</tr>
</tbody>
</table>

1 Problem Description

The problem consists of a T-beam section, as shown in Fig. 1. The section is designed for shear, the shear between web and flanges of T-sections is considered and the required reinforcement is determined.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the shear design of T-sections, for the ultimate limit state. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)

![Figure 2: Connection between flange and web in T-sections](image)

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).
3 Model and Results

The T-section, with properties as defined in Table 1, is to be designed for shear, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2]. The structure analysed, consists of a single span beam with a distributed load in gravity direction. The cross-section geometry, as well as the shear cut under consideration can be seen in Fig. 4.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 20/25</td>
<td>( h = 75.0 \text{ cm} )</td>
<td>( P_g = 155 \text{ kN/m} )</td>
</tr>
<tr>
<td>B 500A</td>
<td>( h_f = 15 \text{ cm}, h_w = 60.0 \text{ cm} )</td>
<td>( d_1 = 7.0 \text{ cm} )</td>
</tr>
<tr>
<td></td>
<td>( b_w = 40 \text{ cm} )</td>
<td>( b_{\text{eff},i} = 67.5 \text{ cm}, b_{\text{eff}} = 175 \text{ cm} )</td>
</tr>
</tbody>
</table>
The system with its loading as well as the moment and shear force are shown in Fig. 5. The reference calculation steps [3] are presented in the next section and the results are given in Table 2.

![Figure 5: Loaded Structure, Resulting Moment and Shear Force](image)

<table>
<thead>
<tr>
<th>Table 2: Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>At beam 1001</td>
</tr>
<tr>
<td>$A_{s1}$ [cm$^2$] at $x = 1.0$ m</td>
</tr>
<tr>
<td>$A_{sf} / s_f$ [cm$^2$/m]</td>
</tr>
<tr>
<td>$V_{Rd,c}$ [kN]</td>
</tr>
<tr>
<td>$V_{Rd,max}$ [kN]</td>
</tr>
<tr>
<td>cot $\theta$</td>
</tr>
<tr>
<td>$z$ [cm] at $x = 1.0$ m</td>
</tr>
<tr>
<td>$V_{Ed} = \Delta F_d$ [kN]</td>
</tr>
</tbody>
</table>
4 Design Process

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

$f_{ck} = 25 \text{ MPa}$

$\sigma_{sd} = 456.52 \text{ MPa}$

Design loads

- Design Load for beam 1001, $x=0.0 \text{ m}$:
  \[ M_{Ed,x=0.0} = 0 \text{ kNm} \]

- Design Load for beam 1001, $x=1.0 \text{ m}$:
  \[ M_{Ed,x=1.0} = 697.5 \text{ kNm} \]

Calculating the longitudinal reinforcement:

- For beam 1001, $x=0.0 \text{ m}$
  \[ \mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{0.0 \cdot 10^{-3}}{1.75 \cdot 0.68^2 \cdot 14.17} = 0.00 \]

- For beam 1001, $x=1.0 \text{ m}$
  \[ \mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{697.5 \cdot 10^{-3}}{1.75 \cdot 0.68^2 \cdot 14.17} = 0.0608 \]

$\omega \approx 0.063$, $\xi \approx 0.967$ and $\zeta \approx 0.086$ (interpolated)

\[ A_{s1} = \frac{1}{\sigma_{sd}} \cdot (\omega \cdot b \cdot d \cdot f_{cd} + N_{Ed}) \]

\[ A_{s1} = \frac{1}{456.52} \cdot (0.063 \cdot 1.75 \cdot 0.68 \cdot 14.17) \cdot 100^2 = 23.27 \text{ cm}^2 \]

$z = \zeta \cdot d = 0.967 \cdot 0.68 \text{ m} \approx 65.7 \text{ cm}$

Calculating the shear between flange and web

The shear force is determined by the change of the longitudinal force, at the junction between one side of a flange and the web, in the separated flange:

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1 The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7(2), Fig. 3.8, which can be seen in Fig. 3.

2 The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.
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\[ \Delta F_d = \left( \frac{M_{Ed,x=1.0}}{z} - \frac{M_{Ed,x=0.0}}{z} \right) \cdot \frac{h_f \cdot b_{eff,i}}{h_f \cdot b_{eff}} \]

For beam 1001 (x=0.00 m) \( M_{Ed} = 0.00 \) therefore:

\[ \Delta F_d = \left( \frac{697.5}{0.657} - 0 \right) \cdot \frac{0.675}{1.75} = 409.36 \text{ kN} \]

The longitudinal shear stress \( \nu_{Ed} \) at the junction between one side of a flange and the web is determined by the change of the normal (longitudinal) force in the part of the flange considered, according to:

\[ \nu_{Ed} = \frac{\Delta F_d}{h_f \cdot \Delta x} \]

In our case \( \Delta x = 1.0 \) because the beam length is \( = 1.00 \text{ m} \).

Please note that AQB is outputting the results per length.

\[ \nu_{Ed} = \frac{409.36}{15 \cdot 100} = 0.272 \text{ kN/m}^2 = 2.72 \text{ MPa} \]

Checking the maximum \( \nu_{Rd,max} \) value to prevent crushing of the struts in the flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied:

\[ \nu_{Ed} \leq \nu_{Rd,max} = \nu \cdot f_{cd} \cdot \sin \theta_f \cdot \cos \theta_f \]

\[ \nu_{Rd,max} = \nu \cdot f_{cd} \cdot \sin \theta_f \cdot \cos \theta_f \]

According to DIN EN 1992-1-1, NDP 6.2.4(4):

\[ \nu = \nu_1 \]

\[ \nu_1 = 0.75 \cdot \nu_2 \]

\[ \nu_2 = 1.1 - \frac{f_{ck}}{500} \leq 1.0 \]

\[ \nu_2 = 1.1 - \frac{20}{500} = 1.1 - 0.04 = 1.06 \geq 1.0 \rightarrow \nu_2 = 1.0 \]

\[ \nu_1 = 0.75 \cdot 1.0 = 0.75 \rightarrow \nu = 0.75 \]

The \( \theta \) value is calculated:

\[ V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left( 1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}} \right) \cdot b_w \cdot z \]

\[ b_w \rightarrow h_f, \quad z \rightarrow \Delta x, \quad c = 0.5 \]

\[ V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left( 1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}} \right) \cdot h_f \cdot \Delta x \]

\[ V_{Rd,cc} = 0.5 \cdot 0.48 \cdot 25^{1/3} \cdot \left( 1 - 1.2 \cdot \frac{0}{14.17} \right) \cdot 0.15 \cdot 1.0 \]

\[ V_{Rd,cc} = 0.1052 \text{ MN} = 105.26 \text{ kN} \]
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\[ 1.0 \leq \cot \theta \leq \frac{1.2 + 1.4 \Delta \sigma_{cd} / f_{cd}}{1 - V_{Rd,cc} / V_{Ed}} \leq 3.0 \]

\[ \cot \theta = \frac{1.2}{1 - 105.26/409.36} = 1.619 \]

\[ \tan \theta = \frac{1}{\cot \theta} = \frac{1}{1.619} = 0.619 \Rightarrow \theta = 31.75^\circ \]

\[ V_{Rd,max} = 0.75 \cdot 14.17 \cdot \sin 31.75 \cdot \cos 31.75 = 4.7502 \text{ MPa} \]

\[ V_{Rd,max} = V_{Rd,max} \cdot h_f \cdot \Delta x = 4.7502 \cdot 0.15 \cdot 1.0 = 0.71253 \text{ MN} \]

\[ V_{Rd,max} = 712.53 \text{ kN} \]

**Checking the value** \( V_{Rd,c} \)

If \( V_{Ed} \) is less than or equal to \( V_{Rd,c} = k \cdot f_{cd} \) no extra reinforcement above that for flexure is required.

\[ V_{Rd,c} = k \cdot f_{cd} \]

For concrete C 25/30 \( \rightarrow f_{cd} = 1.02 \text{ MPa} \)

\[ V_{Rd,c} = 0.4 \cdot 1.02 = 0.408 \text{ MPa} \]

\[ V_{Rd,c} = V_{Rd,c} \cdot h_f \cdot \Delta x = 0.0408 \cdot 15 \cdot 100 = 61.2 \text{ kN} \]

- Calculating the necessary transverse reinforcement:

\[ a_{sf} = \frac{V_{Ed} \cdot h_f}{\cot \theta \cdot f_{yd}} \]

\[ a_{sf} = \frac{2.72 \cdot 0.15}{1.619 \cdot 434.78} \cdot 100^2 = 5.79 \text{ cm}^2 \]
5 Conclusion

This example is concerned with the calculation of the shear between web and flanges of T-sections. It has been shown that the results are reproduced with good accuracy.

6 Literature


