SOFiSTiK

Benchmark Example No. 10

Shear between web and flanges of T-sections

SOFiSTiK | 2018

VERiFiCATiON MANUAL
DCE-EN10: Shear between web and flanges of T-sections

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Front Cover

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Overview

Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB
Input file(s):	t-beam_shear_web_flange.dat

1 Problem Description

The problem consists of a T-beam section, as shown in Fig. 1. The cs is designed for shear, the shear between web and flanges of T-sections is considered and the required reinforcement is determined.

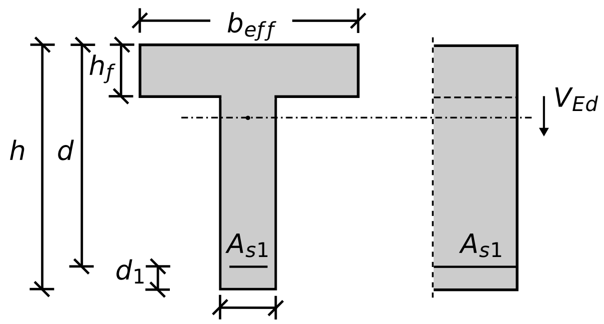

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the shear design of T-sections, for the ultimate limit state. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)

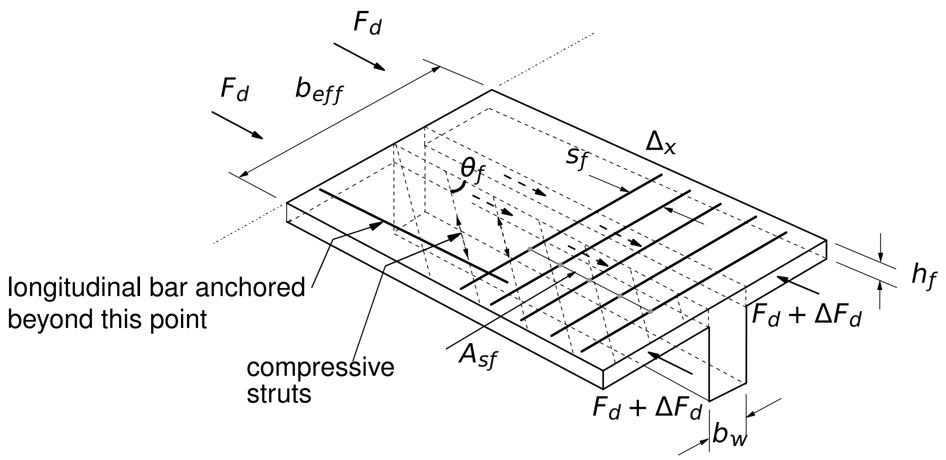

Figure 2: Connection between flange and web in T-sections

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).

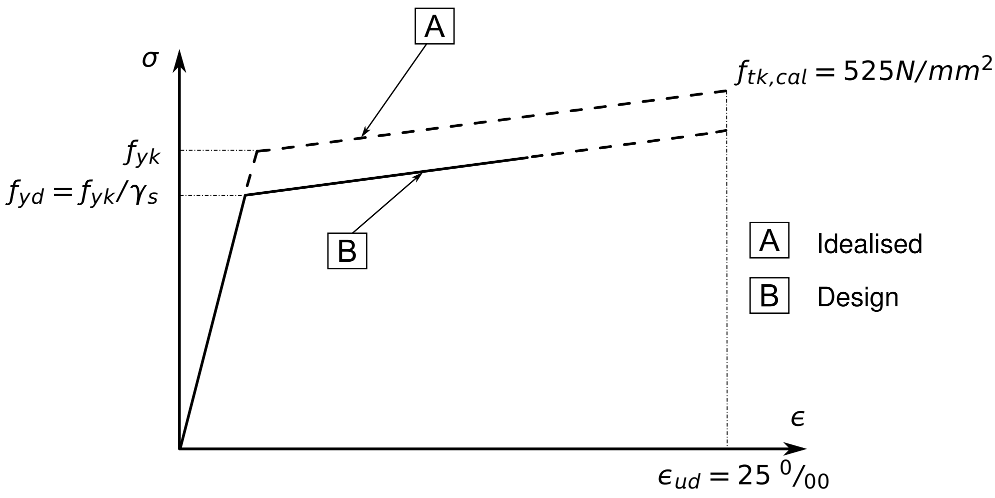

Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The T-section, with properties as defined in Table 1, is to be designed for shear, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2]. The structure analysed, consists of a single span beam with a distributed load in gravity direction. The cross-section geometry, as well as the shear cut under consideration can be seen in Fig. 4.

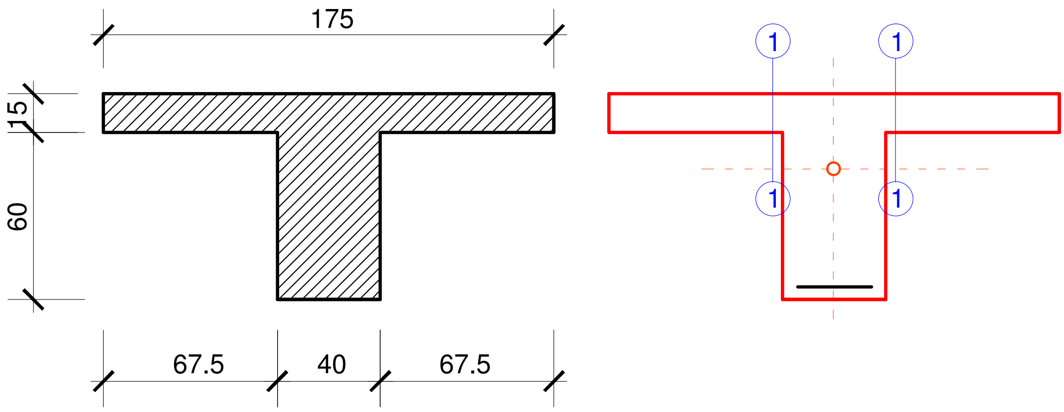

Figure 4: Cross-section Geometry, Properties and Shear Cuts

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\ 20/25$	$h = 75.0\ cm$	$P_g = 155\ kN/m$
$B\ 500A$	$h_f = 15\ cm$, $h_w = 60.0\ cm$	
	$d_1 = 7.0\ cm$	
	$b_w = 40\ cm$	
	$b_{eff,i} = 67.5\ cm$, $b_{eff} = 175\ cm$	

The system with its loading as well as the moment and shear force are shown in Fig. 5. The reference calculation steps [3] are presented in the next section and the results are given in Table 2.

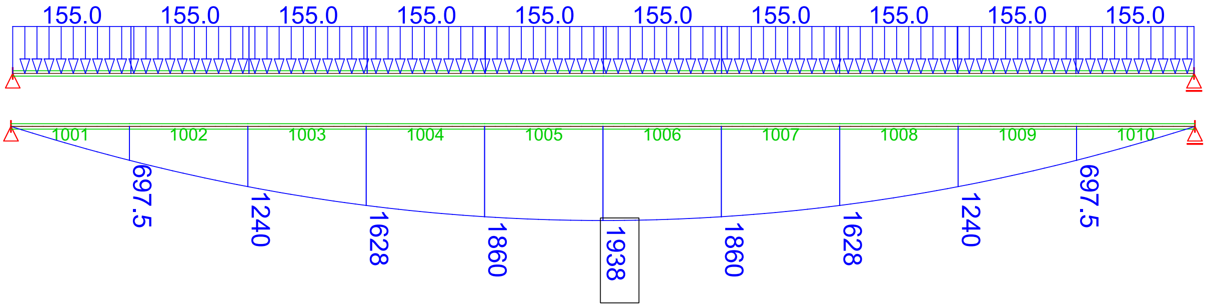

Figure 5: Loaded Structure, Resulting Moment and Shear Force

Table 2: Results

At beam 1001	SOF.	Ref.
A_{s1} $[cm^2]$ at $x = 1.0\ m$	23.25	23.27
$A_{sf}\ /\ s_f\ \ [cm^2/m]$	5.72	5.79
$V_{Rd,c}\ \ [kN]$	61.05	61.2
$V_{Rd,max}\ \ [kN]$	711.68	712.53
$\cot\theta$	1.62	1.619
$z\ [cm]$ at $x = 1.0\ m$	65.72	65.7
$V_{Ed} = \Delta F_d\ \ [kN]$	403.65	409.36

4 Design Process[1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

Tab. 3.1: Strength for concrete

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

3.2.2: (3)P: yield strength $f_{yk} = 500$ MPa

3.2.7: (2), Fig. 3.8

$f_{ck} = 25\ MPa$
$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 25/1.5 = 14.17\ MPa$

$f_{yk} = 500\ MPa$
$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

$\sigma_{sd} = 456.52\ MPa$

Design loads

- Design Load for beam 1001, x=0.0 m:

$M_{Ed,x=0.0\ m} = 0.0\ kNm$

- Design Load for beam 1001, x=1.0 m:

$M_{Ed,x=1.0\ m} = 697.5\ kNm$

Calculating the longitudinal reinforcement:

- For beam 1001, x=0.0 m

$\mu = 0 \rightarrow A_{s1} = 0$

$$\mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{0.0 \cdot 10^{-3}}{1.75 \cdot 0.68^2 \cdot 14.17} = 0.00$$

- For beam 1001, x=1.0 m

$$\mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{697.5 \cdot 10^{-3}}{1.75 \cdot 0.68^2 \cdot 14.17} = 0.0608$$

Tab. 9.2 [4]: ω—Table for up to $C50/60$ without compression reinforcement

$\omega \approx 0.063$, $\zeta \approx 0.967$ and $\xi \approx 0.086$ (interpolated)

$N_{Ed} = 0$

$$A_{s1} = \frac{1}{\sigma_{sd}} \cdot (\omega \cdot b \cdot d \cdot f_{cd} + N_{Ed})$$

$$A_{s1} = \frac{1}{456.52} \cdot (0.063 \cdot 1.75 \cdot 0.68 \cdot 14.17) \cdot 100^2 = 23.27\ cm^2$$

$$z = \zeta \cdot d = 0.967 \cdot 0.68\ m \approx 65.7\ cm$$

Calculating the shear between flange and web

6.2.4 (3): Eq. 6.20

The shear force, is determined by the change of the longitudinal force, at the junction between one side of a flange and the web, in the separated flange:

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

[2] The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

$$\Delta F_d = \left(\frac{M_{Ed,x=1.0}}{z} - \frac{M_{Ed,x=0.0}}{z}\right) \cdot \frac{h_f \cdot b_{eff,i}}{h_f \cdot b_{eff}}$$

For beam 1001 (x=0.00 m) → $M_{Ed} = 0.00$ therefore:

$$\Delta F_d = \left(\frac{697.5}{0.657} - 0\right) \cdot \frac{0.675}{1.75} = 409.36\ kN$$

In AQB output $\Delta F_d = T[kN/m]$

The longitudinal shear stress v_{Ed} at the junction between one side of a flange and the web is determined by the change of the normal (longitudinal) force in the part of the flange considered, according to:

$$v_{Ed} = \frac{\Delta Fd}{h_f \cdot \Delta x}$$

DIN EN 1992, 6.2.4 (3),Eq. 6.20: h_f is the thickness of flange at the junctions, Δx is the length under consideration, ΔF_d is the change of the normal force in the flange over the length Δx

In our case $\Delta x = 1.0$ because the beam length is = 1.00 m.

Please note that AQB is outputting the results per length.

$$v_{Ed} = \frac{409.36}{15 \cdot 100} = 0.272\ kN/m^2 = 2.72\ MPa$$

In AQB output $v_{Ed} = \tau - V$

Checking the maximum $v_{Rd,max}$ value to prevent crushing of the struts in the flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied:

$$v_{Ed} \leq v_{Rd,max} = \nu \cdot f_{cd} \cdot sin\theta_f \cdot cos\theta_f$$

DIN EN 1992, 6.2.4 (4),Eq. 6.22

$$v_{Rd,max} = \nu \cdot f_{cd} \cdot sin\theta_f \cdot cos\theta_f$$

According to DIN EN 1992-1-1, NDP 6.2.4(4):

$$\nu = \nu_1$$

$$\nu_1 = 0.75 \cdot \nu_2$$

DIN EN 1992, NDP 6.2.3 (3),Eq. 6.22

$$\nu_2 = 1.1 - \frac{f_{ck}}{500} \leq 1.0$$

$$\nu_2 = 1.1 - \frac{20}{500} = 1.1 - 0.04 = 1.06 \geq 1.0 \rightarrow \nu_2 = 1.0$$

$$\nu_1 = 0.75 \cdot 1.0 = 0.75 \rightarrow \nu = 0.75$$

The θ value is calculated:

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left(1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}}\right) \cdot b_w \cdot z$$

DIN EN 1992, NDP 6.2.3 (2),Eq. NA.6.7b: $\sigma_{cd} = N_{Ed}/A_C$, c=0.5

$$b_w \rightarrow h_f, \quad z \rightarrow \Delta x, \quad c = 0.5$$

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left(1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}}\right) \cdot h_f \cdot \Delta x$$

$$V_{Rd,cc} = 0.5 \cdot 0.48 \cdot 25^{1/3} \cdot \left(1 - 1.2 \cdot \frac{0}{14.17}\right) \cdot 0.15 \cdot 1.0$$

$$V_{Rd,cc} = 0.1052\ MN = 105.26\ kN$$

DIN EN 1992, NDP 6.2.3 (2),Eq. NA.6.7a

$$1.0 \leq cot\theta \leq \frac{1.2 + 1.4 \cdot \Delta\sigma_{cd}/f_{cd}}{1 - V_{Rd,cc}/V_{Ed}} \leq 3.0$$

$$cot\theta = \frac{1.2}{1 - 105.26/409.36} = 1.619$$

$$\tan\theta = \frac{1}{cot\theta} = \frac{1}{1.619} = 0.619 \rightarrow \theta = 31.75°$$

$$v_{Rd,max} = 0.75 \cdot 14.17 \cdot sin31.75 \cdot cos31.75 = 4.7502\ MPa$$

$$V_{Rd,max} = v_{Rd,max} \cdot h_f \cdot \Delta x = 4.7502 \cdot 0.15 \cdot 1.0 = 0.71253\ MN$$

$$V_{Rd,max} = 712.53\ kN$$

Checking the value $v_{Rd,c}$

If v_{Ed} is less than or equal to $v_{Rd,c} = k \cdot f_{ctd}$ no extra reinforcement above that for flexure is required.

$$v_{Rd,c} = k \cdot f_{ctd}$$

For concrete C 25/30 $\rightarrow f_{ctd} = 1.02\ MPa$

$$v_{Rd,c} = 0.4 \cdot 1.02 = 0.408\ MPa$$

$$V_{Rd,c} = v_{Rd,c} \cdot h_f \cdot \Delta x = 0.0408 \cdot 15 \cdot 100 = 61.2\ kN$$

- Calculating the necessary transverse reinforcement:

DIN EN 1992, 6.2.4 (4),Eq. 6.21

$$a_{sf} = \frac{v_{Ed} \cdot h_f}{cot\theta_f \cdot f_{yd}}$$

$$a_{sf} = \frac{2.72 \cdot 0.15}{1.619 \cdot 434.78} \cdot 100^2 = 5.79\ cm^2$$

5 Conclusion

This example is concerned with the calculation of the shear between web and flanges of T-sections. It has been shown that the results are reproduced with good accuracy.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau*. Ernst & Sohn. Deutschen Beton- und Bautechnik-Verein E.V. 2011.

[4] K. Holschemacher, T. Müller, and F. Lobisch. *Bemessungshilfsmittel für Betonbauteile nach Eurocode 2 Bauingenieure*. 3rd. Ernst & Sohn, 2012.