Benchmark Example No. 12

Crack width calculation of reinforced beam acc. DIN EN 1992-1-1

SOFiSTiK | 2018
DCE-EN12: Crack width calculation of reinforced beam acc. DIN EN 1992-1-1

VERiFiCATiON MANUAL, Version 2018-14
Software Version: SOFiSTiK 2018

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VERiFiCATiON MANUAL

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The manual and the program have been thoroughly checked for errors. However, SOFiSTiK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.

Front Cover

Overview

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<th>Design Code Family(s):</th>
<th>DIN</th>
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<td>Design Code(s):</td>
<td>DIN EN 1992-1-1</td>
</tr>
<tr>
<td>Module(s):</td>
<td>AQB</td>
</tr>
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<td>Input file(s):</td>
<td>crack_widths.dat</td>
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1 Problem Description

The problem consists of a rectangular section, asymmetrically reinforced, as shown in Fig. 1. Different loading conditions are examined, always consisting of a bending moment $M_{Ed}$, and in addition with or without a compressive or tensile axial force $N_{Ed}$. The crack width is determined.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the control of crack widths. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Basic assumptions for calculation of crack widths (Section 7.3.2, 7.3.3, 7.3.4)

![Figure 2: Stress and Strain Distributions in the Design of Cross-sections](image)

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).
3 Model and Results

The rectangular cross-section, with properties as defined in Table 1, is to be designed for crack width, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1] [2]. The calculation steps with different loading conditions and calculated with different sections of DIN EN 1992-1-1:2004 are presented below and the results are given in Table 2.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 25/30</td>
<td>$h = 100.0 \text{ cm}$</td>
<td>$N_{Ed} = 0$ or $\pm 300 \text{ kN}$</td>
</tr>
<tr>
<td>B 500A</td>
<td>$d = 96.0 \text{ cm}$</td>
<td>$M_{Ed} = 562.5 \text{ kNm}$</td>
</tr>
<tr>
<td></td>
<td>$b = 30.0 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\phi_s = 25.0 \text{ mm}$, $A_s = 24.50 \text{ cm}^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\phi_{s'} = 12.0 \text{ mm}$, $A_{s'} = 2.26 \text{ cm}^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$w_k = 0.3 \text{ mm}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th>Case</th>
<th>Load $M_{Ed}, N_{Ed}$</th>
<th>$A_s$ given $[\text{ cm}^2]$</th>
<th>Result $A_{s,requ} [\text{ cm}^2]$</th>
<th>SOF.</th>
<th>Ref.</th>
<th>$\sigma_s [\text{ MPa}]$</th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$M_{Ed}, N_{Ed} = 0$</td>
<td>24.50</td>
<td>$A_{s,requ}$</td>
<td>6.93</td>
<td>6.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_s [\text{ MPa}]$</td>
<td>207.85</td>
<td>207.85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>$M_{Ed}, N_{Ed} = 300$</td>
<td>24.50</td>
<td>$A_{s,requ}$</td>
<td>10.39</td>
<td>10.39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>$M_{Ed}, N_{Ed} = -300$</td>
<td>24.50</td>
<td>$A_{s,requ}$</td>
<td>4.04</td>
<td>4.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>$M_{Ed}, N_{Ed} = 0$</td>
<td>24.50</td>
<td>$A_{s}$</td>
<td>passed with given reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_s [\text{ MPa}]$</td>
<td>440.57</td>
<td>440.53</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>$M_{Ed}, N_{Ed} = 0$</td>
<td>12.0</td>
<td>$A_{s}$</td>
<td>not passed with given reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_s [\text{ MPa}]$</td>
<td>436.30</td>
<td>436.28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>$M_{Ed}, N_{Ed} = 0$</td>
<td>24.50</td>
<td>$w_k [\text{ mm}]$</td>
<td>0.13</td>
<td>0.13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4 Design Process

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:

Material:

Concrete: \( \gamma_c = 1.50 \)

Steel: \( \gamma_s = 1.15 \)

\( f_{ck} = 25 \text{ MPa} \)

\( f_{cd} = \alpha_{cd} \cdot f_{ck} / \gamma_c = 0.85 \cdot 25 / 1.5 = 14.17 \text{ MPa} \)

\( f_{yk} = 500 \text{ MPa} \)

\( f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78 \text{ MPa} \)

Design Load:

\( M_{Ed} = 562.5 \text{ kNm} \)

\( N_{Ed} = 0.0 \text{ or } \pm 300 \text{ kN} \)

**Minimum reinforcement areas**

- **Case I**: \( M_{Ed}, N_{Ed} = 0.0 \)

\[
A_{s,min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}
\]

\( f_{ct,eff} = f_{ctm} \geq 3.0 \text{ MPa} \)

\( f_{ct,eff} = 2.6 < 3.0 \text{ MPa} \rightarrow f_{ct,eff} = 3.0 \text{ MPa} \)

\( A_{ct} = b \cdot h_{eff} = 0.3 \cdot 0.5 = 0.15 \text{ m}^2 \)

\( k_c = 0.4 \cdot \left[ 1 - \frac{\sigma_c}{k_1 \cdot (h/h^*) \cdot f_{ct,eff}} \right] \leq 1 \)

\( \sigma_c = N_{Ed} / (b \cdot h) = 0.0 \rightarrow k_c = 0.4 \)

\( k = 0.8 \text{ for } h \leq 300 \)

\( \phi_s = \phi_s^* \cdot f_{ct,eff} / 2.9 \text{ N/mm}^2 \)

\( \phi_s^* = 25 \cdot 2.9 / 3.0 = 24.16667 \)

---

1 The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8.

2 The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.
\[ \sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 / \phi_s^*} = 207.846 \text{ MPa} \]

\[ A_{s,\text{req}} = 0.4 \cdot 0.8 \cdot 3.0 \cdot 0.15 \cdot 10^4 / 207.846 = 6.928 \text{ cm}^2 \]

- **Case II**: \( N_{Ed} = 300 \text{ kN} \)
  \[ A_{s,\min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,\text{eff}} \cdot A_c \]
  \[ f_{ct,\text{eff}} = f_{ctm} \geq 3.0 \text{ MPa} \]
  \[ f_{ct,\text{eff}} = 2.6 < 3.0 \text{ MPa} \rightarrow f_{ct,\text{eff}} = 3.0 \text{ MPa} \]

\[ A_c = b \cdot h_{\text{eff}} = 0.15 \text{ m}^2 \]

\[ \sigma_c = \frac{N_{Ed}}{b \cdot h} = \frac{300 \cdot 10^3}{300 \cdot 1000} = 1.0 \text{ MPa} \]

\[ k = 0.8 \text{ for } h \leq 300 \]

\[ \phi_s = \phi_s^* \cdot f_{ct,\text{eff}} / 2.9 \text{ N/mm}^2 \]

\[ \phi_s^* = 25 \cdot 2.9 / 3.0 = 24.16667 \]

\[ \sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 / \phi_s^*} = 207.846 \text{ MPa} \]

\[ k_c = 0.4 \cdot \left[ 1 - \frac{\sigma_c}{k_1 \cdot (h/h^*) \cdot f_{ct,\text{eff}}} \right] \leq 1 \]

\[ k_1 = 2 \cdot h^* / (3 \cdot h) = 2 / 3 \]

\[ k_c = 0.4 \cdot \left[ 1 + \frac{1.0}{(2 / 3) \cdot 1.3 \cdot 3.0} \right] = 0.6 \leq 1 \]

\[ A_{s,\text{req}} = 0.6 \cdot 0.8 \cdot 3.0 \cdot 0.15 \cdot 10^4 / 207.846 = 10.39 \text{ cm}^2 \]

- **Case III**: \( N_{Ed} = -300 \text{ kN} \)
  \[ A_{s,\min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,\text{eff}} \cdot A_c \]
  \[ f_{ct,\text{eff}} = f_{ctm} \geq 3.0 \text{ MPa} \]
  \[ f_{ct,\text{eff}} = 2.6 < 3.0 \text{ MPa} \rightarrow f_{ct,\text{eff}} = 3.0 \text{ MPa} \]

\[ A_c = b \cdot h_{\text{eff}} \]

The height of the tensile zone is determined through the stresses:

\[ \sigma_c = \frac{N_{Ed}}{b \cdot h} = \frac{300 \cdot 10^3}{300 \cdot 1000} = 1.0 \text{ MPa} \]

\[ \sigma_u = f_{ct,\text{eff}} = 3.0 \text{ MPa} \]

\[ \Rightarrow h_{\text{eff}} = \frac{3.0 \cdot 50}{3.0 + 1.0} = 37.5 \text{ cm} \]

\[ A_c = 0.3 \cdot 0.375 = 0.1125 \text{ m}^2 \]

\[ k = 0.8 \text{ for } h \leq 300 \]
\[ \phi_s = \phi_s^* \cdot f_{ct, eff} / 2.9 \text{ N/mm}^2 \]
\[ \phi_s^* = 25 \cdot 2.9 / 3.0 = 24.1667 \]
\[ \sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 / \phi_s^*} = 207.846 \text{ MPa} \]
\[ k_c = 0.4 \cdot \left[ 1 - \frac{\sigma_c}{k_1 \cdot (h/h^*) \cdot f_{ct, eff}} \right] \leq 1 \]
\[ k_1 = 1.5 \]
\[ k_c = 0.4 \cdot \left[ 1 - \frac{1.0}{1.5 \cdot 1.3} \right] = 0.3111 \leq 1 \]
\[ A_{s, requ} = 0.3111 \cdot 0.8 \cdot 3.0 \cdot 0.1125 \cdot 10^4 / 207.846 = 4.04 \text{ cm}^2 \]

**Control of cracking without direct calculation**

- **Case IV:** \( N_{Ed} = 0 \)

\( f_{ct, eff} = f_{ctm} \geq 3.0 \text{ MPa} \)

\[ f_{ct, eff} = 2.6 < 3.0 \text{ MPa} \rightarrow f_{ct, eff} = 3.0 \text{ MPa} \]

\[ \phi_s = \phi_s^* \cdot \frac{\sigma_s \cdot A_s}{4(h - d) \cdot b \cdot 2.9} \geq \phi_s^* \cdot \frac{f_{ct, eff}}{2.9} \]

\[ \phi_s = 25 \text{ mm} = \phi_s^* \cdot \frac{264.06 \cdot 24.50}{4(100 - 96) \cdot 30 \cdot 2.9} = \phi_s^* \cdot 4.6476 \]

\[ \rightarrow \phi_s^* = 5.3791 \text{ mm} \]

\[ \sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 / \phi_s^*} = 440.53 \text{ MPa} \]

\[ \sigma_s = 264.06 < 440.53 \text{ MPa} \]

which corresponds to the value calculated in SOFiSTIK [ssr]

\[ \rightarrow \text{crack width control is passed with given reinforcement.} \]

In case the usage factor becomes 1.0 then the stresses \( \sigma_s \) are equal, as it can be seen in Case V below.

and \( \phi_s = 25 \text{ mm} > \phi_s^* \cdot \frac{f_{ct, eff}}{2.9} = 5.3791 \cdot \frac{3.0}{2.9} = 5.5646 \)

also control the steel stress with respect to the calculated strains

\[ \epsilon_s = 0.440 + 1.913 \cdot (0.50 - 0.04) = 1.31998 \]

\[ \rightarrow \sigma_s = 0.00131998 \cdot 200000 = 264.0 \text{ MPa} \]

or

from Tab. 7.2DE and for \( \sigma_s = 264.04 \approx 260.0 \text{ MPa} \)

\[ \rightarrow \phi_s^* = 15 \text{ mm} \rightarrow \phi_s = \phi_s^* \cdot \frac{\sigma_s \cdot A_s}{4(h - d) \cdot b \cdot 2.9} \geq \phi_s^* \cdot \frac{f_{ct, eff}}{2.9} \]
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\[ \phi_s = 15 \cdot \frac{264.06 \cdot 24.50}{4(100 - 96) \cdot 30 \cdot 2.9} = 69.69 \text{ mm} > 25 \text{ mm} \]

→ crack width control is passed with given reinforcement.

• Case V: \( N_{Ed} = 0.0, A_s = 12.0 \text{ cm}^2 \)

In this case, the prescribed reinforcement is decreased from \( A_s = 24.5 \text{ cm}^2 \) to \( A_s = 12.0 \text{ cm}^2 \) in order to examine a case where the crack width control is not passed with the given reinforcement.

\[ f_{ct, eff} = f_{ctm} \geq 3.0 \text{ MPa} \]

\[ f_{ct, eff} = 2.6 < 3.0 \text{ MPa} \rightarrow f_{ct, eff} = 3.0 \text{ MPa} \]

from Tab. 7.2DE and for \( \sigma_s = 509.15 \approx 510.0 \text{ MPa} \)

\[ \phi_s \approx 3.9 \text{ mm} \rightarrow \phi_s = \phi^* \cdot \frac{\sigma_s \cdot A_s}{4(h - d) \cdot b \cdot 2.9} \]

\[ \phi_s = 3.9 \cdot \frac{509.15 \cdot 12.0}{4(100 - 96) \cdot 30 \cdot 2.9} = 17.12 \text{ mm} < 25 \text{ mm} \]

→ crack width control is not passed with given reinforcement.

→ start increasing reinforcement in order to be in the limits of admissible steel stresses

from Tab. 7.2DE and for \( \sigma_s = 436.43 \text{ MPa} \)

\[ \phi_s \approx 5.6 \text{ mm} \rightarrow \phi_s = \phi^* \cdot \frac{\sigma_s \cdot A_s}{4(h - d) \cdot b \cdot 2.9} \]

\[ \phi_s = 5.6 \cdot \frac{436.43 \cdot 14.54}{4(100 - 96) \cdot 30 \cdot 2.9} = 25.45 \text{ mm} \geq 25 \text{ mm} \]

→ crack width control passed with additional reinforcement.

If we now input as prescribed reinforcement the reinforcement that is calculated in order to pass crack control, i.e. \( A_s = 14.54 \text{ cm}^2 \) we get a steel stress of \( \sigma_s = 436.36 \text{ MPa} \) which gives

\[ \phi_s = 25 \text{ mm} = \phi^* \cdot \frac{436.36 \cdot 14.54}{4(100 - 96) \cdot 30 \cdot 2.9} \]

\[ \phi^* = 5.4849 \text{ mm} \]

\[ \sigma_s = \sqrt{w_k \cdot 3.48 \cdot 10^6 / \phi^*} \]

\[ \sigma_s = \sqrt{0.3 \cdot 3.48 \cdot 10^6 / 5.4849} = 436.28 \text{ MPa} \]

Here we can notice that the stresses are equal leading to a usage factor of 1.0

Control of cracking with direct calculation
• Case VI: $N_{Ed} = 0$

\[
\alpha_e = E_s / E_{cm} = 200000 / 31476 = 6.354
\]

\[
\rho_{p, eff} = (A_s + \frac{\xi_1^2 \cdot A_p'}{A_c, eff}) / A_c, eff
\]

\[
A_c, eff = h_{c, eff} \cdot b
\]

\[
h / d_1 = 100/4 = 25.00 \rightarrow h_{c, eff} / d_1 = 3.25
\]

\[
A_c, eff = (3.25 \cdot 4) \cdot 30 = 13 \cdot 30 = 390 \, \text{cm}^2
\]

\[
A_p' = 0.0 \, \text{cm}^2
\]

\[
\rho_{p, eff} = 24.50 / 390 = 0.06282
\]

\[
\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_1 \cdot f_{ct, eff} \cdot (1 + \alpha_e \cdot \rho_{p, eff})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}
\]

\[
f_{ct, eff} = f_{ctm} = 0.30 \cdot f_{ck}^{2/3} = 2.565 \approx 2.6 \, \text{MPa}
\]

\[
f_{ct, eff} = 2.6 < 3.0 \, \text{MPa} \rightarrow f_{ct, eff} = 3.0 \, \text{MPa}
\]

\[
\epsilon_{sm} - \epsilon_{cm} = \frac{264.06 - 0.4 \cdot \frac{2.565}{0.06282} \cdot (1 + 6.354 \cdot 0.06282)}{200000} = 1.2060 \cdot 10^{-3} > 0.6 \cdot \frac{264.06}{200000} = 0.79218 \cdot 10^{-3}
\]

\[
s_{r, max} = \frac{\phi}{3.6 \cdot \rho_{p, eff}} \leq \frac{\sigma_s \cdot \phi}{3.6 \cdot f_{ct, eff}}
\]

\[
s_{r, max} = \frac{25}{3.6 \cdot 0.06282} = 110.545 \, \text{mm}
\]

\[
s_{r, max} \leq \frac{\sigma_s \cdot \phi}{3.6 \cdot f_{ct, eff}} = 611.25 \, \text{mm}
\]

\[
w_k = s_{r, max} \cdot (\epsilon_{sm} - \epsilon_{cm}) = 110.545 \cdot 1.2060 \cdot 10^{-3}
\]

\[
w_k = 0.133 < 0.3 \, \text{mm} \rightarrow \text{Check for crack width passed with given reinforcements}
\]

**Stress limitation**

\[
\sigma_{max, t} = k_3 \cdot f_yk
\]

\[
\sigma_{max, t} = 0.80 \cdot 500 \, \text{MPa}
\]

\[
\sigma_{max, t} = 400 \, \text{MPa}
\]
5 Conclusion

This example shows the calculation of crack widths. Various ways of reference calculations are demonstrated, in order to compare the SOFiSTiK results to. It has been shown that the results are reproduced with excellent accuracy.

6 Literature
