Benchmark Example No. 21

Real Creep and Shrinkage Calculation of a T-Beam Prestressed CS

SOFiSTiK | 2018

VERiFiCATiON MANUAL
DCE-EN21: Real Creep and Shrinkage Calculation of a T-Beam Prestressed CS

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Front Cover

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Overview	
Design Code Family(s):	EN
Design Code(s):	EN 1992-1-1
Module(s):	CSM
Input file(s):	real_creep_shrinkage.dat

1 Problem Description

The problem consists of a simply supported beam with a T-Beam cross-section of prestressed concrete, as shown in Fig. 1. The nodal displacement is calculated considering the effects of real creep and shrinkage, also the usage of custom (experimental) creep and shrinkage parameters is verified, the custom (experimental) parameter is taken from fib Model Code 2010 [1].

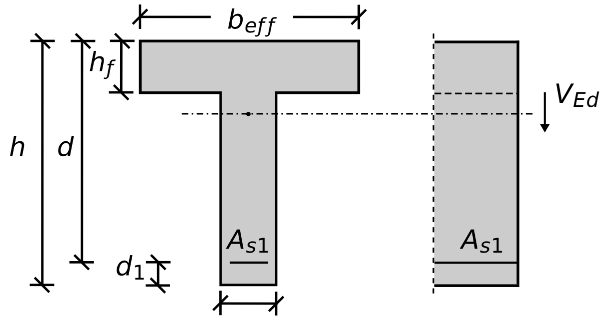

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the calculation of creep and shrinkage on a prestressed concrete cs, subject to horizontal prestressing force. The content of this problem is covered by the following parts of EN 1992-1-1:2004 [2]:

- Creep and Shrinkage (Section $3.1.4$)
- Annex B: Creep and Shrinkage (Section $B.1$, $B.2$)

The time dependant displacements are calculated by multiplying the length of the beam with the creep (ϵ_{cc}) and shrinkage (ϵ_{cs}) strain:

- the creep deformation of concrete is calculated according to (EN 1992-1-1, 3.1.4, Eq. 3.6)
- the total shrinkage strain is calculated according to (EN 1992-1-1, 3.1.4, Eq. 3.8)

3 Model and Results

The benchmark 21 is here to show the effects of real creep on a prestressed concrete simply supported beam. The analysed system can be seen in Fig. 4 with properties as defined in Table 1. The tendon geometry is simplified as much as possible and modelled as a horizontal force, therefore tendons are not subject of this benchmark. The beam consists of a T-Beam cs and is loaded with a horizontal prestressing force from time $t_1 = 100$ days to time $t_2 = 300$ days. The self-weight is neglected. A calculation of the creep and shrinkage is performed in the middle of the span with respect to EN 1992-1-1:2004 [2]. The calculation steps are presented below and the results are given in Table 2 for the calculation with CSM. For calculating the real creep and shrinkage (RCRE) an equivalent loading is used, see Fig. 2 and Fig. 3.

The time steps for the calculation are:
$t_0 = 7$ days, $t_1 = 100$ days, $t_2 = 300$ days, $t_\infty = 30$ years

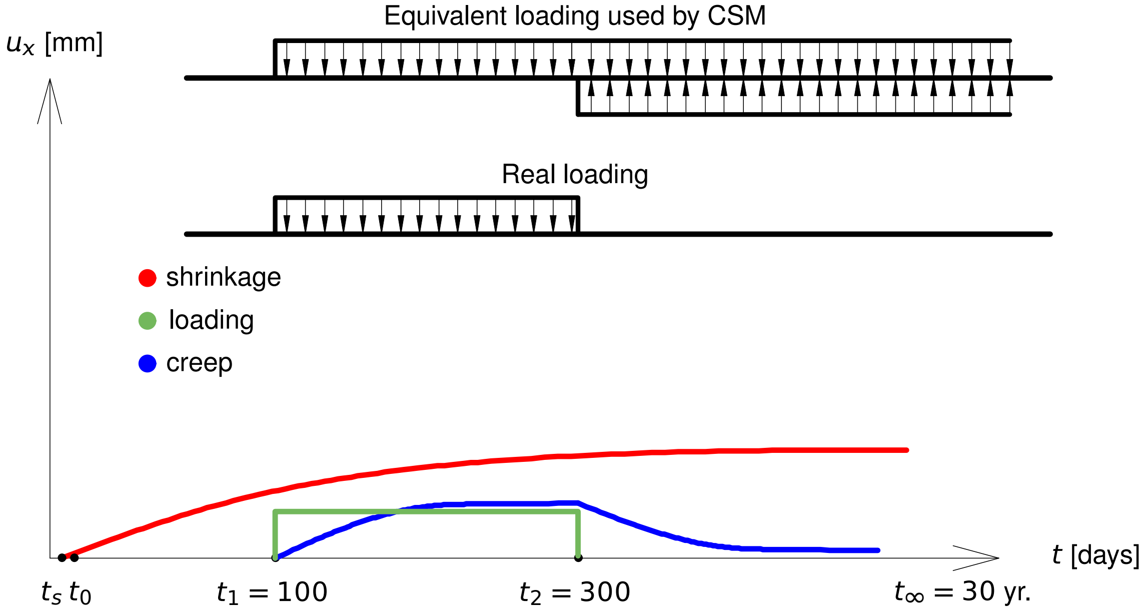

Figure 2: Creep, shrinkage and loading displacements

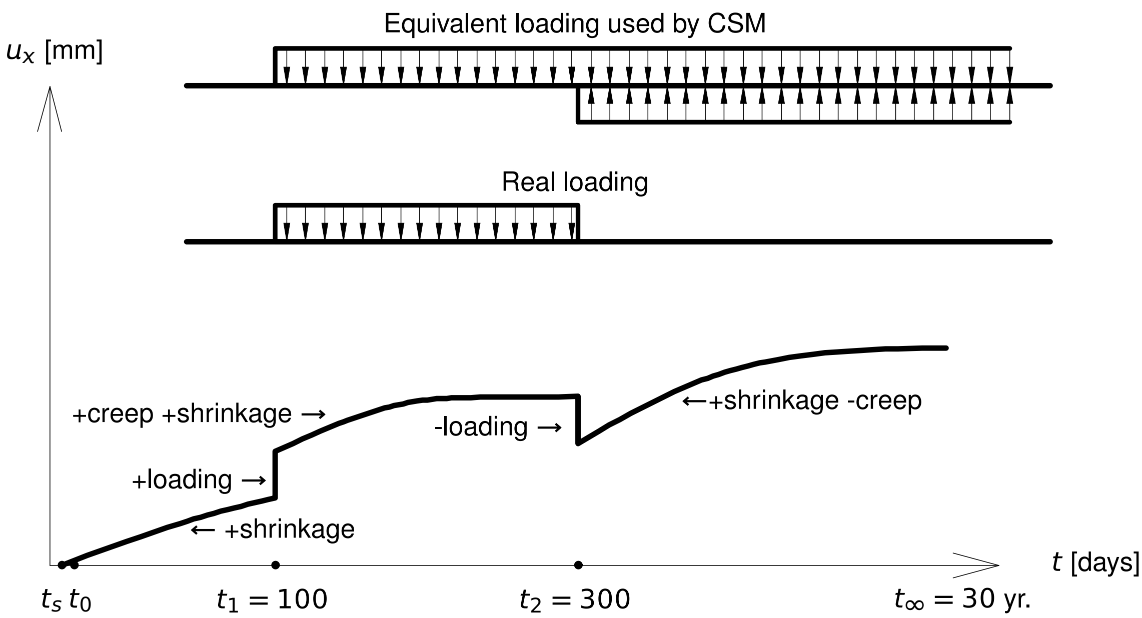

Figure 3: Equivalent loading and displacement for real creep and shrinkage (RCRE)

The benchmark contains next calculation steps:

1. Calculating the shrinkage displacements before loading.

2. Calculating the displacements when loading occurs at time $t_1 = 100$ days.
3. Calculating the displacements (creep and shrinkage) at time before the loading is inactive ($t_2 \approx 300$ and $t_2 < 300$ days).
4. Calculating the displacements at time when the loading is inactive ($t_2 \approx 300$ and $t_2 > 300$ days).
5. Calculating the displacements at time $t_3 = 30$ years.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading (at $x = 10\ m$)	Time
$C\ 35/45$	$h = 120\ cm$	$N_p = -900.0\ kN$	$t_0 = 7$ days
$Y\ 1770$	$b_{eff} = 280.0\ cm$		$t_s = 3$ days
$RH = 80$	$h_f = 40\ cm$		
	$b_w = 40\ cm$		
	$L = 20.0\ m$		

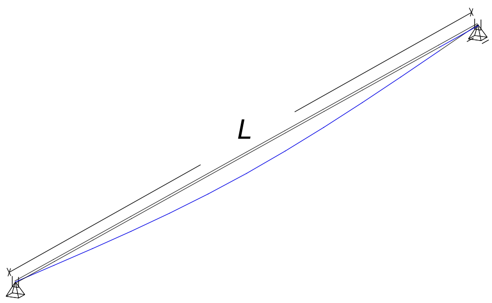

Figure 4: Simply Supported Beam

Table 2: Results

Result	CSM [mm]	Ref [mm].
Δl_{4015}	-0.69	-0.688
Δl_{4020}	-0.849	-0.8431
Δl_{4025}	-1.455	-1.45314
Δl_{4030}	-1.298	-1.29814
Δl_{4035}	-2.166	-2.08

4 Design Process[1]

Design with respect to EN 1992-1-1:2004 [2]:[2]

Material:

3.1: Concrete

Concrete: $C\ 35/45$

3.1.2: Tab. 3.1: E_{cm}, f_{ck} and f_{cm} for $C\ 35/45$

$E_{cm} = 34077\ N/mm^2$

$f_{ck} = 35\ N/mm^2$

$f_{cm} = 43\ N/mm^2$

3.3: Prestressing Steel

Prestressing Steel: $Y\ 1770$

Load Actions:

Self weight per length is neglected: $\gamma = 0\ kN/m$ (to simplify the example as much as possible)

At $x = 10.0\ m$ middle of the span:

$N_{Ed} = -900\ kN$
$A = 280 \cdot 40 + 60 \cdot 80 = 16000\ cm^2$

Calculation of stresses at $x = 10.0\ m$ midspan:

σ_c stress in concrete

$$\sigma_c = \frac{N_{Ed}}{A} = \frac{-900}{16000} = -0.05625\ kN/cm^2 = -0.5625\ N/mm^2$$

1) Calculating the shrinkage displacements before loading

• Calculating creep:

According to EN 1992-1-1 the creep deformation of concrete for a constant compressive stress σ_c applied at a concrete age t_0 is given by:

$\epsilon_{cc} = \phi(t, t_0) \cdot (\sigma_c / E_{cs})$

Because $\sigma_c = 0$ (before loading), creep deformation is neglected and $\epsilon_{cc} = 0$.

• Calculating shrinkage:

t_0 minimum age of concrete for loading
t_s age of concrete at start of drying shrinkage
t age of concrete at the moment considered

$t_0 = 7$ days

$t_s = 3$ days

$t = 100$ days

$t_{eff} = t - t_0 = 100 - 7 = 93$ days

3.1.4 (6): Eq. 3.8: ϵ_{cs} total shrinkage strain

$\epsilon_{cs} = \epsilon_{cd} + \epsilon_{ca}$

3.1.4 (6): Eq. 3.9: ϵ_{cd} drying shrinkage strain

$\epsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \epsilon_{cd,0}$

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [2] 3.3.6: Fig. 3.10

[2] The sections mentioned in the margins refer to EN 1992-1-1:2004 [2], [3], unless otherwise specified.

The development of the drying shrinkage strain in time is strongly depends on $\beta_{ds}(t, t_s)$ factor. SOFiSTiK accounts not only for the age at start of drying t_s but also for the influence of the age of the prestressing t_0. Therefore, the calculation of factor β_{ds} reads:

$$\beta_{ds} = \beta_{ds}(t, t_s) - \beta_{ds}(t_0, t_s)$$

3.1.4 (6): Eq. 3.10: β_{ds}

$$\beta_{ds} = \frac{(t - t_s)}{(t - t_s) + 0.04 \cdot \sqrt{h_0^3}} - \frac{(t_0 - t_s)}{(t_0 - t_s) + 0.04 \cdot \sqrt{h_0^3}}$$

3.1.4 (6): h_0 the notional size (mm) of the cs $h_0 = 2A_c/u = 500\ mm$

$$\beta_{ds} = \frac{(100 - 3)}{(100 - 3) + 0.04 \cdot \sqrt{400^3}} - \frac{(7 - 3)}{(7 - 3) + 0.04 \cdot \sqrt{400^3}}$$

$$\beta_{ds} = 0.232 - 0.01235 = 0.22026$$

$$k_h = 0.725 \text{ for } h_0 = 400\ mm$$

3.1.4 (6): Tab. 3.3: k_h coefficient depending on h_0

$$\epsilon_{cd,0} = 0.85\left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right] \cdot 10^{-6} \cdot \beta_{RH}$$

Annex B.2 (1): Eq. B.11: $\epsilon_{cd,0}$ basic drying shrinkage strain

$$\beta_{RH} = 1.55\left[1 - \left(\frac{RH}{RH_0}\right)^3\right] = 1.55\left[1 - \left(\frac{80}{100}\right)^3\right] = 0.7564$$

Annex B.2 (1): Eq. B.12: β_{RH}
RH the ambient relative humidity (%)

$$\epsilon_{cd,0} = 0.85\left[(220 + 110 \cdot 4) \cdot \exp\left(-0.12 \cdot \frac{43}{10}\right)\right] \cdot 10^{-6} \cdot 0.7564$$

Annex B.2 (1): $\alpha_{ds1}, \alpha_{ds1}$ coefficients depending on type of cement.
For class N $\alpha_{ds1} = 4$, $\alpha_{ds2} = 0.12$

$$\epsilon_{cd,0} = 2.533 \cdot 10^{-4}$$

$$\epsilon_{cd} = \beta_{ds} \cdot k_h \cdot \epsilon_{cd,0}$$

$$\epsilon_{cd} = 0.22026 \cdot 0.725 \cdot 2.533 \cdot 10^{-4} = -4.04 \cdot 10^{-5}$$

Drying shrinkage:

$$\epsilon_{cd} = -4.04 \cdot 10^{-5}$$

$$\epsilon_{ca}(t) = \beta_{as}(t) \cdot \epsilon_{ca}(\infty)$$

3.1.4 (6): Eq. 3.11: ϵ_{ca} autogenous shrinkage strain

$$\epsilon_{ca}(\infty) = 2.5\,(f_{ck} - 10) \cdot 10^{-6} = 2.5 \cdot (35 - 10) \cdot 10^{-6}$$

3.1.4 (6): Eq. 3.12: $\epsilon_{ca}(\infty)$

$$\epsilon_{ca}(\infty) = 6.25 \cdot 10^{-5} = 0.0625\ ‰$$

Proportionally to $\beta_{ds}(t, ts)$, SOFiSTiK calculates factor β_{as} as follows:

$$\beta_{as} = \beta_{as}(t) - \beta_{as}(t_0)$$

3.1.4 (6): Eq. 3.13: β_{as}

$$\beta_{as} = 1 - e^{-0.2 \cdot \sqrt{t}} - \left(1 - e^{-0.2 \cdot \sqrt{t_0}}\right) = e^{-0.2 \cdot \sqrt{t_0}} - e^{-0.2 \cdot \sqrt{t}}$$

$$\beta_{as} = 0.4537$$

$$\epsilon_{ca}(t) = \beta_{as}(t) \cdot \epsilon_{ca}(\infty)$$

$$\epsilon_{ca}(t) = 0.45377 \cdot 6.25 \cdot 10^{-5}$$

Autogenous shrinkage:

$\epsilon_{ca}(t) = -2.84 \cdot 10^{-5}$

Total shrinkage:

$\epsilon_{cs} = \epsilon_{ca} + \epsilon_{cd}$

ϵ absolute shrinkage strain
negative sign to declare losses

$\epsilon_{cs} = -2.84 \cdot 10^{-5} + (-4.04) \cdot 10^{-5} = -6.881 \cdot 10^{-5}$

Calculating displacement:

$\Delta l_{1,cs} = \epsilon_{cs} \cdot L/2$

$\Delta l_{1,cs} = -6.881 \cdot 10^{-5} \cdot 10000\ mm$

$\Delta l_{1,cs} = -0.6881\ mm$

2) Calculate displacement when loading occurs at time $t_1 = 100$ days at $x = 10.0\ m$ midspan

E_{cs} calculated "ideal" cross section modulus of elasticity for concrete and reinforcement steel

$\sigma_c = E_{cs} \cdot \epsilon$

$$E_{cs} = E_{cm} + \frac{A_s}{A_c} \cdot E_s$$

$$E_{cs} = 3407.7 + \frac{178.568}{16000 - 178.568} \cdot 20000$$

$E_{cs} = 3407.7 + 225.729$

$E_{cs} = 3633.42\ kN/cm^2 = 36334.29\ N/mm^2$

$$\epsilon = \frac{\sigma_c}{E_{cs}} = \frac{-0.5625}{36334.29} = -1.55 \cdot 10^{-5}$$

$$\epsilon = \frac{\Delta l_2}{l} \rightarrow \Delta l_2 = \epsilon \cdot L/2$$

$\Delta l_2 = -1.55 \cdot 10^{-5} \cdot 10000\ mm = -0.155\ mm$

3) Calculating the displacement (creep and shrinkage) at time before the loading is inactive ($t_2 \approx 300$ and $t_2 < 300$ days)

t_0 minimun age of concrete for loading
t_s age of concrete at start of drying shrinkage
t age of concrete at the moment considered

$t_0 = 100$ days

$t_s = 3$ days

$t = 300$ days

$t_{eff} = t - t_0 = 300 - 100 = 200$ days

- Calculating shrinkage:

3.1.4 (6): Eq. 3.8: ϵ_{cs} total shrinkage strain

$\epsilon_{cs} = \epsilon_{cd} + \epsilon_{ca}$

3.1.4 (6): Eq. 3.9: ϵ_{cd} drying shrinkage strain

$\epsilon_{cd}(t) = \beta_{ds}(t, ts) \cdot k_h \cdot \epsilon_{cd,0}$

The development of the drying shrinkage strain in time is strongly depends on $\beta_{ds}(t, ts)$ factor. SOFiSTiK accounts not only for the age at start of drying t_s but also for the influence of the age of the prestressing

t_0. Therefore, the calculation of factor β_{ds} reads:

$$\beta_{ds} = \beta_{ds}(t, t_s) - \beta_{ds}(t_0, t_s)$$

3.1.4 (6): Eq. 3.10: β_{ds}

$$\beta_{ds} = \frac{(t - t_s)}{(t - t_s) + 0.04 \cdot \sqrt{h_0^3}} - \frac{(t_0 - t_s)}{(t_0 - t_s) + 0.04 \cdot \sqrt{h_0^3}}$$

3.1.4 (6): h_0 the notional size (mm) of the cs $h_0 = 2A_c/u = 500\ mm$

$$\beta_{ds} = \frac{(300 - 3)}{(300 - 3) + 0.04 \cdot \sqrt{400^3}} - \frac{(100 - 3)}{(100 - 3) + 0.04 \cdot \sqrt{400^3}}$$

$$\beta_{ds} = 0.2487$$

$$k_h = 0.725 \text{ for } h_0 = 400\ mm$$

3.1.4 (6): Tab. 3.3: k_h coefficient depending on h_0

$$\epsilon_{cd,0} = 0.85 \left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}} \right) \right] \cdot 10^{-6} \cdot \beta_{RH}$$

Annex B.2 (1): Eq. B.11: $\epsilon_{cd,0}$ basic drying shrinkage strain

$$\beta_{RH} = 1.55 \left[1 - \left(\frac{RH}{RH_0} \right)^3 \right] = 1.55 \left[1 - \left(\frac{80}{100} \right)^3 \right] = 0.7564$$

Annex B.2 (1): Eq. B.12: β_{RH}
RH the ambient relative humidity (%)

$$\epsilon_{cd,0} = 0.85 \left[(220 + 110 \cdot 4) \cdot \exp\left(-0.12 \cdot \frac{43}{10} \right) \right] \cdot 10^{-6} \cdot 0.7564$$

Annex B.2 (1): $\alpha_{ds1}, \alpha_{ds1}$ coefficients depending on type of cement.
For class N $\alpha_{ds1} = 4$, $\alpha_{ds2} = 0.12$

$$\epsilon_{cd,0} = 2.533 \cdot 10^{-4}$$

$$\epsilon_{cd} = 0.24874 \cdot 0.725 \cdot 2.533 \cdot 10^{-4} = -4.57 \cdot 10^{-5}$$

Drying shrinkage:

$$\epsilon_{cd} = -4.57 \cdot 10^{-5}$$

$$\epsilon_{ca}(t) = \beta_{as}(t) \cdot \epsilon_{ca}(\infty)$$

3.1.4 (6): Eq. 3.11: ϵ_{ca} autogenous shrinkage strain

$$\epsilon_{ca}(\infty) = 2.5 \cdot (f_{ck} - 10) \cdot 10^{-6} = 2.5\,(35 - 10) \cdot 10^{-6}$$

3.1.4 (6): Eq. 3.12: $\epsilon_{ca}(\infty)$

$$\epsilon_{ca}(\infty) = 6.25 \cdot 10^{-5} = 0.0625\ ‰$$

Proportionally to $\beta_{ds}(t, ts)$, SOFiSTiK calculates factor β_{as} as follows:

$$\beta_{as} = \beta_{as}(t) - \beta_{as}(t_0)$$

3.1.4 (6): Eq. 3.13: β_{as}

$$\beta_{as} = 1 - e^{-0.2 \cdot \sqrt{t}} - \left(1 - e^{-0.2 \cdot \sqrt{t_0}} \right) = e^{-0.2 \cdot \sqrt{t_0}} - e^{-0.2 \cdot \sqrt{t}}$$

$$\beta_{as} = 0.104$$

$$\epsilon_{ca}(t) = \beta_{as}(t) \cdot \epsilon_{ca}(\infty)$$

$$\epsilon_{ca}(t) = 0.1040 \cdot 6.25 \cdot 10^{-5}$$

Autogenous shrinkage:

$$\epsilon_{ca}(t) = -6.502136 \cdot 10^{-6}$$

Total shrinkage:

$$\epsilon_{cs} = \epsilon_{ca} + \epsilon_{cd}$$

$$\epsilon_{cs} = -6.502136 \cdot 10^{-6} + (-4.57) \cdot 10^{-5}$$

$$\epsilon_{cs} = -5.218 \cdot 10^{-5}$$

ϵ absolute shrinkage strain
negative sign to declare losses

Calculating displacement:

$$\Delta l_{3,cs} = \epsilon_{cs} \cdot L/2$$

$$\Delta l_{3,cs} = -5.218 \cdot 10^{-5} \cdot 10000\ mm$$

$$\Delta l_{3,cs} = -0.5218\ mm$$

- Calculating creep:

Annex B.1 (1): Eq. B.1: $\phi(t, t_0)$ creep coefficient

$$\phi(t, t_0) = \phi_0 \cdot \beta_c(t, t_0)$$

Annex B.1 (1): Eq. B.2: ϕ_0 notional creep coefficient

$$\phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

Annex B.1 (1): Eq. B.3: ϕ_{RH} factor for effect of relative humidity on creep

$$\phi_{RH} = \left[1 + \frac{1 - RH/100}{0.1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1\right] \cdot \alpha_2$$

Annex B.1 (1): Eq. B.4: $\beta(f_{cm})$ factor for effect of concrete strength on creep

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = 16.8/\sqrt{43} = 2.562$$

Annex B.1 (1): Eq. B.8c: α_1, α_2, α_3 coefficients to consider influence of concrete strength

$$\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7} = 0.8658 \leq 1$$

$$\alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2} = 0.9597 \leq 1$$

$$\alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5} = 0.9022 \leq 1$$

$$\phi_{RH} = \left[1 + \frac{1 - 80/100}{0.1 \cdot \sqrt[3]{400}} \cdot 0.8658\right] \cdot 0.9597 = 1.1852$$

Annex B.1 (1): Eq. B.5: $\beta(t_0)$ factor for effect of concrete age at loading on creep

$$\beta(t_0) = \frac{1}{\left(0.1 + t_0^{0.20}\right)}$$

Annex B.1 (2): Eq. B.9: $t_{0,T}$ temperature adjusted age of concrete at loading adjusted according to expression B.10

$$t_0 = t_{0,T} \cdot \left(\frac{9}{2 + t_{0,T}^{1.2}} + 1\right)^{\alpha} \geq 0.5$$

Annex B.1 (3): Eq. B.10: t_T temperature adjusted concrete age which replaces t in the corresponding equations

$$t_T = \sum_{i=1}^{n} e^{-(4000/[273+T(\Delta t_i)]-13.65)} \cdot \Delta t_i$$

$$t_{0,T} = 100 \cdot e^{-(4000/[273+20]-13.65)} = 100 \cdot 1.0 = 100.0$$

Annex B.1 (2): Eq. B.9: α a power which depends on type of cement
For class N $\alpha = 0$

$$\Rightarrow t_0 = 100 \cdot \left(\frac{9}{2 + 100^{1.2}} + 1\right)^{0} = 100$$

$$\beta(t_0) = \frac{1}{\left(0.1 + 100^{0.20}\right)} = 0.383$$

The coefficient to describe the development of creep with time after loading can be calculated according to EN 1992-1-1, Eq. B.7.:

$$\beta_c(t,t_0) = \left[\frac{(t-t_0)}{(\beta_H + t - t_0)}\right]^{0.3}$$

Annex B.1 (1): Eq. B.7: $\beta_c(t,t_0)$ coefficient to describe the development of creep with time after loading

In SOFiSTiK it is possible to modify and use custom creep and shrinkage parameters, for more details see AQUA and AQB Manual. In our case we are using the equation from fib Model Code 2010[1] (to verify and show the MEXT feature in SOFiSTiK):

$$\beta_c(t,t_0) = \left[\frac{(t-t_0)}{(\beta_H + t - t_0)}\right]^{\gamma(t_0)}$$

fib Model Code 2010; Eq. 5.1-71a

where:

$$\gamma(t_0) = \frac{1}{2.3 + \frac{3.5}{\sqrt{t_0}}} = \frac{1}{2.3 + \frac{3.5}{\sqrt{100}}} = \frac{1}{2.65} = 0.3773$$

fib Model Code 2010; Eq. 5.1-71b

With `MEXT NO 1 EIGE VAL 0.3773` the exponent 0.3 can be modified to 0.3773.

$$\beta_c(t,t_0) = \left[\frac{(t-t_0)}{(\beta_H + t - t_0)}\right]^{0.3773}$$

$$\beta_H = 1.5 \cdot \left[1 + (0.012 \cdot RH)^{18}\right] \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3$$

$$\beta_H = 1.5 \cdot \left[1 + (0.012 \cdot 80)^{18}\right] \cdot 400 + 250 \cdot 0.9022$$

$$\beta_H = 1113.31 \leq 1500 \cdot 0.9022 = 1353.30$$

Annex B.1 (1): Eq. B.7: $\beta_c(t,t_0)$ coefficient to describe the development of creep with time after loading

Annex B.1 (1): Eq. B.8: β_H coefficient depending on relative humidity and notional member size

$$\Rightarrow \beta_c(t,t_0) = 0.4916$$

$$\phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

$$\phi_0 = 1.1852 \cdot 2.5619 \cdot 0.383 = 1.1629$$

$$\phi(t,t_0) = \phi_0 \cdot \beta_c(t,t_0)$$

$$\phi(t,t_0) = 1.1629 \cdot 0.4916 = 0.57$$

$$\phi_{eff}(t,t_0) = 0.57/1.05 = 0.5445$$

According to EN, the creep value is related to the tangent Young's modulus E_c, where E_c being defined as $1.05 \cdot E_{cm}$. To account for this, SOFiSTiK adopts this scaling for the computed creep coefficient (in SOFiSTiK, all computations are consistently based on E_{cm}).

Annex B.1 (3): The values of $\phi(t,t_0)$ given above should be associated with the tangent modulus E_c

3.1.4 (2): The values of the creep coefficient, $\phi(t,t_0)$ is related to E_c, the tangent modulus, which may be taken as $1.05 \cdot E_{cm}$

Calculating the displacement:

$$\epsilon_{cc}(t,t_0) = \phi(t,t_0) \cdot \frac{\sigma_c}{E_{cs}}$$

$$\epsilon_{cc}(t,t_0) = 0.57 \cdot \frac{-0.5625}{36334.29} = -8.82430 \cdot 10^{-6}$$

$$\epsilon = \frac{\Delta l}{l} \rightarrow \Delta l_{3,cc} = \epsilon_{cc} \cdot L/2$$

$$\Delta l_{3,cc} = -8.82430 \cdot 10^{-6} \cdot 10000\ mm = -0.08824\ mm$$

4) Calculating the displacement at time when the loading is inactive ($t_2 \approx 300$ and $t_2 > 300$ days).

At this step the loading disappears therefore:

$$\Delta l_4 = -\Delta l_2 = 0.155\ mm$$

5) Calculating the displacement at time $t_3 = 30$ years.

t_0 minimun age of concrete for loading
t_s age of concrete at start of drying shrinkage
t age of concrete at the moment considered

$$t_0 = 300 \text{ days}$$

$$t_s = 3 \text{ days}$$

$$t = 11250 \text{ days}$$

$$t_{eff} = t - t_0 = 11250 - 300 = 11950\ days$$

$$\rightarrow 11950/365 = 30\ years$$

- Calculating shrinkage:

3.1.4 (6): Eq. 3.8: ϵ_{cs} total shrinkage strain

$$\epsilon_{cs} = \epsilon_{cd} + \epsilon_{ca}$$

3.1.4 (6): Eq. 3.9: ϵ_{cd} drying shrinkage strain

$$\epsilon_{cd}(t) = \beta_{ds}(t, ts) \cdot k_h \cdot \epsilon_{cd,0}$$

The development of the drying shrinkage strain in time is strongly depends on $\beta_{ds}(t, ts)$ factor. SOFiSTiK accounts not only for the age at start of drying t_s but also for the influence of the age of the prestressing t_0. Therefore, the calculation of factor β_{ds} reads:

3.1.4 (6): Eq. 3.10: β_{ds}

$$\beta_{ds} = \beta_{ds}(t, t_s) - \beta_{ds}(t_0, t_s)$$

3.1.4 (6): h_0 the notional size (mm) of the cs $h_0 = 2A_c/u = 500\ mm$

$$\beta_{ds} = \frac{(t - t_s)}{(t - t_s) + 0.04 \cdot \sqrt{h_0^3}} - \frac{(t_0 - t_s)}{(t_0 - t_s) + 0.04 \cdot \sqrt{h_0^3}}$$

$$\beta_{ds} = \frac{(11250 - 3)}{(11250 - 3) + 0.04 \cdot \sqrt{400^3}} - \frac{(300 - 3)}{(300 - 3) + 0.04 \cdot \sqrt{400^3}}$$

$$\beta_{ds} = 0.49097$$

3.1.4 (6): Tab. 3.3: k_h coefficient depending on h_0

$$k_h = 0.725 \text{ for } h_0 = 400\ mm$$

Annex B.2 (1): Eq. B.11: $\epsilon_{cd,0}$ basic drying shrinkage strain

$$\epsilon_{cd,0} = 0.85 \left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}} \right) \right] \cdot 10^{-6} \cdot \beta_{RH}$$

Annex B.2 (1): Eq. B.12: β_{RH}
RH the ambient relative humidity (%)

$$\beta_{RH} = 1.55 \left[1 - \left(\frac{RH}{RH_0} \right)^3 \right] = 1.55 \left[1 - \left(\frac{80}{100} \right)^3 \right] = 0.7564$$

Annex B.2 (1): $\alpha_{ds1}, \alpha_{ds1}$ coefficients depending on type of cement.
For class N $\alpha_{ds1} = 4$, $\alpha_{ds2} = 0.12$

$$\epsilon_{cd,0} = 0.85 \left[(220 + 110 \cdot 4) \cdot \exp\left(-0.12 \cdot \frac{43}{10} \right) \right] \cdot 10^{-6} \cdot 0.7564$$

$$\epsilon_{cd,0} = 2.533 \cdot 10^{-4}$$

$$\epsilon_{cd} = \beta_{ds}(t, t_s) \cdot k_h \cdot \epsilon_{cd,0}$$

$$\epsilon_{cd} = 0.49097 \cdot 0.725 \cdot 2.533 \cdot 10^{-4} = -9.02 \cdot 10^{-5}$$

Drying shrinkage:

$$\epsilon_{cd} = -9.02 \cdot 10^{-5}$$

$$\epsilon_{ca}(t) = \beta_{as}(t) \cdot \epsilon_{ca}(\infty)$$

3.1.4 (6): Eq. 3.11: ϵ_{ca} autogenous shrinkage strain

$$\epsilon_{ca}(\infty) = 2.5 \cdot (f_{ck} - 10) \cdot 10^{-6} = 2.5\,(35 - 10) \cdot 10^{-6}$$

3.1.4 (6): Eq. 3.12: $\epsilon_{ca}(\infty)$

$$\epsilon_{ca}(\infty) = 6.25 \cdot 10^{-5} = 0.0625\ ‰$$

Proportionally to $\beta_{ds}(t, ts)$, SOFiSTiK calculates factor β_{as} as follows:

$$\beta_{as} = \beta_{as}(t) - \beta_{as}(t_0)$$

3.1.4 (6): Eq. 3.13: β_{as}

$$\beta_{as} = 1 - e^{-0.2 \cdot \sqrt{t}} - \left(1 - e^{-0.2 \cdot \sqrt{t_0}}\right) = e^{-0.2 \cdot \sqrt{t_0}} - e^{-0.2 \cdot \sqrt{t}}$$

$$\beta_{as} = 0.03130$$

$$\epsilon_{ca}(t) = \beta_{as}(t) \cdot \epsilon_{ca}(\infty)$$

$$\epsilon_{ca}(t) = 0.03130 \cdot 6.25 \cdot 10^{-5}$$

Autogenous shrinkage:

$$\epsilon_{ca}(t) = -1.95632 \cdot 10^{-6}$$

Total shrinkage:

$$\epsilon_{cs} = \epsilon_{ca} + \epsilon_{cd}$$

$$\epsilon_{cs} = -1.95632 \cdot 10^{-6} + (-9.02) \cdot 10^{-5}$$

$$\epsilon_{cs} = -9.212 \cdot 10^{-5}$$

ϵ absolute shrinkage strain negative sign to declare losses

Calculating displacement:

$$\Delta l_{5,cs} = \epsilon_{cs} \cdot L/2$$

$$\Delta l_{5,cs} = -9.212 \cdot 10^{-5} \cdot 10000\ mm$$

$$\Delta l_{5,cs} = -0.9212\ mm$$

- Calculating creep:

$$\phi(t, t_0) = \phi_0 \cdot \beta_c(t, t_0)$$

Annex B.1 (1): Eq. B.1: $\phi(t, t_0)$ creep coefficient

$$\phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

Annex B.1 (1): Eq. B.2: ϕ_0 notional creep coefficient

$$\phi_{RH} = \left[1 + \frac{1 - RH/100}{0.1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1\right] \cdot \alpha_2$$

Annex B.1 (1): Eq. B.3: ϕ_{RH} factor for effect of relative humidity on creep

Annex B.1 (1): Eq. B.4: $\beta(f_{cm})$ factor for effect of concrete strength on creep

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = 16.8/\sqrt{43} = 2.562$$

Annex B.1 (1): Eq. B.8c: α_1, α_2, α_3 coefficients to consider influence of concrete strength

$$\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7} = 0.8658 \leq 1$$

$$\alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2} = 0.9597 \leq 1$$

$$\alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5} = 0.9022 \leq 1$$

$$\phi_{RH} = \left[1 + \frac{1 - 80/100}{0.1 \cdot \sqrt[3]{400}} \cdot 0.8658\right] \cdot 0.9597 = 1.1852$$

Annex B.1 (1): Eq. B.5: $\beta(t_0)$ factor for effect of concrete age at loading on creep

$$\beta(t_0) = \frac{1}{\left(0.1 + t_0^{0.20}\right)}$$

Annex B.1 (2): Eq. B.9: $t_{0,T}$ temperature adjusted age of concrete at loading adjusted according to expression B.10

$$t_0 = t_{0,T} \cdot \left(\frac{9}{2 + t_{0,T}^{1.2}} + 1\right)^{\alpha} \geq 0.5$$

Annex B.1 (3): Eq. B.10: t_T temperature adjusted concrete age which replaces t in the corresponding equations

$$t_T = \sum_{i=1}^{n} e^{-(4000/[273+T(\Delta t_i)]-13.65)} \cdot \Delta t_i$$

$$t_{0,T} = 300 \cdot e^{-(4000/[273+20]-13.65)} = 300 \cdot 1.0 = 300.0$$

Annex B.1 (2): Eq. B.9: α a power which depends on type of cement
For class N $\alpha = 0$

$$\Rightarrow t_0 = 300 \cdot \left(\frac{9}{2 + 300^{1.2}} + 1\right)^{0} = 300$$

$$\beta(t_0) = \frac{1}{\left(0.1 + 300^{0.20}\right)} = 0.3097$$

The coefficient to describe the development of creep with time after loading can be calculated according to EN 1992-1-1, Eq. B.7.:

Annex B.1 (1): Eq. B.7: $\beta_c(t, t_0)$ coefficient to describe the development of creep with time after loading

$$\beta_c(t, t_0) = \left[\frac{(t - t_0)}{(\beta_H + t - t_0)}\right]^{0.3}$$

In SOFiSTiK it is possible to modify and use custom creep and shrinkage parameters, for more details see AQUA and AQB Manual. In our case we are using the equation from fib Model Code 2010[1] (to verify and show the MEXT feature in SOFiSTiK).

With `MEXT NO 1 EIGE VAL 0.3773` the exponent 0.3 can be modified to 0.3773.

Annex B.1 (1): Eq. B.7: $\beta_c(t, t_0)$ coefficient to describe the development of creep with time after loading

$$\beta_c(t, t_0) = \left[\frac{(t - t_0)}{(\beta_H + t - t_0)}\right]^{0.3773}$$

Annex B.1 (1): Eq. B.8: β_H coefficient depending on relative humidity and notional member size

$$\beta_H = 1.5 \cdot \left[1 + (0.012 \cdot RH)^{18}\right] \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3$$

$$\beta_H = 1.5 \cdot \left[1 + (0.012 \cdot 80)^{18}\right] \cdot 400 + 250 \cdot 0.9022$$

$$\beta_H = 1113.31 \leq 1500 \cdot 0.9022 = 1353.30$$

$$\Rightarrow \beta_c(t, t_0) = 0.9641$$

$\phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$

$\phi_0 = 1.1852 \cdot 2.5619 \cdot 0.3097 = 0.94067$

$\phi(t, t_0) = \phi_0 \cdot \beta_c(t, t_0)$

$\phi(t, t_0) = 0.94067 \cdot 0.9641 = 0.91$

$\phi_{eff}(t, t_0) = 0.91/1.05 = 0.8637$

According to EN, the creep value is related to the tangent Young's modulus E_c, where E_c being defined as $1.05 \cdot E_{cm}$. To account for this, SOFiSTiK adopts this scaling for the computed creep coefficient (in SOFiSTiK, all computations are consistently based on E_{cm}).

Annex B.1 (3): The values of $\phi(t, t_0)$ given above should be associated with the tangent modulus E_c

3.1.4 (2): The values of the creep coefficient, $\phi(t, t_0)$ is related to E_c, the tangent modulus, which may be taken as $1.05 \cdot E_{cm}$

$$\epsilon_{cc}(t, t_0) = \phi(t, t_0) \cdot \frac{\sigma_c}{E_{cs}}$$

$$\epsilon_{cc}(t, t_0) = 0.91 \cdot \frac{-0.5625}{36334.29} = -1.4087 \cdot 10^{-5}$$

$$\epsilon = \frac{\Delta l}{l} \rightarrow \Delta l_{5,cc} = \epsilon_{cc} \cdot L/2$$

$$\Delta l_{5,cc} = -1.4087 \cdot 10^{-5} \cdot 10000\ mm = -0.1408\ mm$$

CALCULATING THE DISPLACEMENT:

- 4010 stripping concrete

$\Delta l_{4010} = 0$ mm

- 4015 K creep step

$\Delta l_{4015} = \Delta l_{1,cs}$

$\Delta l_{4015} = -0.688$ mm

- 4020 Start loading A

$\Delta l_{4020} = \Delta l_{1,cs} + \Delta l_2$

$\Delta l_{4020} = -0.6881 - 0.155$

$\Delta l_{4020} = -0.8431$ mm

- 4025 K creep step

$\Delta l_{4025} = \Delta l_{1,cs} + \Delta l_2 + \Delta l_{3,cs} + \Delta_{3,cc}$

$\Delta l_{4025} = -0.6881 - 0.155 - 0.08824 - 0.5218$

$\Delta l_{4025} = -1.45314$ mm

- 4030 Stop loading A

$\Delta l_{4030} = \Delta l_{1,cs} + \Delta l_2 + \Delta l_{3,cs} + \Delta l_{3,cc} - \Delta l_4$

$\Delta l_{4030} = -0.6881 - 0.155 - 0.08824 - 0.5218 + 0.155$

$\Delta l_{4030} = -1.29814$ mm

- 4035 K creep step

$\Delta l_{4035} = \Delta l_{4030} + \Delta l_{5,cs} - \Delta l_{5,cc}$

$\Delta l_{4035} = -1.29814 - 0.9212 + 0.140$

$\Delta l_{4035} \approx -2.08$ mm

5 Conclusion

This example shows the calculation of the time dependent displacements due to creep and shrinkage. It has been shown that the results are in very good agreement with the reference solution.

6 Literature

[1] fib Model Code 2010. *fib Model Code for Concrete Structures 2010*. International Federation for Structural Concrete (fib). 2010.

[2] *EN 1992-1-1: Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2004.

[3] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.