Benchmark Example No. 23

Stress Relaxation of Prestressing Steel - DIN EN 1992-1-1

SOFiSTiK | 2018

VERiFiCATiON MANUAL
DCE-EN23: Stress Relaxation of Prestressing Steel - DIN EN 1992-1-1

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Front Cover

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Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB, AQUA
Input file(s):	relaxation_din_en1992.dat

1 Problem Description

The problem consists of a simply supported beam with a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The time dependent losses are calculated, considering the reduction of stress caused by the deformation of prestressing steel due to steel-relaxation, under the permanent loads.

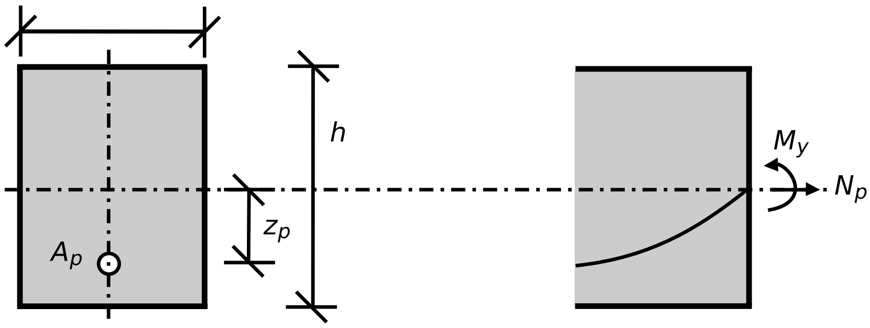

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the calculation of relaxation losses on a prestressed concrete cs, subject to bending and prestress force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1] [2]:

- Properties (Section $3.3.2$)
- Annex D: Detailed calculation method for prestressing steel relaxation losses (Section $D.1$)
- Strength (Section $3.3.3$)
- DIN-HB, "Zulassung Spannstahl" [3]

In this Benchmark the stress loss due to relaxation will be examined, creep and shrinkage losses are neglected and disabled.

3 Model and Results

Benchmark 17 is here extended for the case of steel relaxation losses developing on a prestressed concrete simply supported beam. The analysed system can be seen in Fig. 2, with properties as defined in Table 1. Further information about the tendon geometry and prestressing can be found in benchmark 17. The beam consists of a rectangular cs and is prestressed and loaded with its own weight. A calculation of the relaxation stress losses is performed in the middle of the span with respect to DIN EN 1992-1-1:2004 [1] [2]. The calculation steps are presented below and the results are given in Table 2 for the calculation with AQB.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading (at $x = 10\ m$)	Time
$C\ 35/45$	$h = 100.0\ cm$	$M_g = 1250\ kNm$	$t = 1000\ h$
$Y\ 1770$	$b = 100.0\ cm$	$N_p = -3653.0\ kN$	
$RH = 80$	$L = 20.0\ m$		
	$A_p = 28.5\ cm^2$		

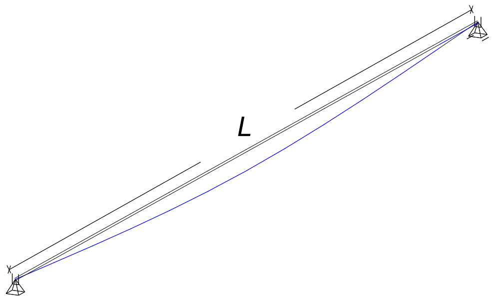

Figure 2: Simply Supported Beam

Table 2: Results

Result	AQB	Ref.
$\Delta\sigma_{pr,total}$	$29.15\ MPa$	$29.15\ MPa$
ΔP_{pr}	$83.071\ kN$	$83.071\ kN$
$\frac{\Delta P_{pr}}{P_0}[\%]$	2.274%	2.274%

4 Design Process[1]

Design with respect to DIN EN 1992-1-1:2004 [1] [2]

- Material:

Concrete: $C\ 35/45$

3.1: Concrete

$E_{cm} = 34077\ N/mm^2$

3.1.2: Tab. 3.1: E_{cm}, f_{ck} and f_{cm} for $C\ 35/45$

$f_{ck} = 35\ N/mm^2$

$f_{cm} = 43\ N/mm^2$

Prestressing Steel: $Y\ 1770$

3.3: Prestressing Steel

$E_p = 195000\ N/mm^2$

3.3.6 (3): E_p for wires

$f_{pk} = 1770\ N/mm^2$

3.3.2, 3.3.3: f_{pk} Characteristic tensile strength of prestressing steel

$f_{p0,1k} = 1520\ N/mm^2$

3.3.2, 3.3.3: $f_{p0,1k}$ 0.1% proof-stress of prestressing steel, yield strength

Prestressing system: $BBV\ L19\ 150\ mm^2$

19 wires with area of $150\ mm^2$ each, giving a total of $A_p = 28.5\ cm^2$

- Cross-section:

$A_c = 1.0 \cdot 1.0 = 1\ m^2$

Diameter of duct $\phi_{duct} = 97\ mm$

Ratio $\alpha_{E,p} = E_p\ /\ E_{cm} = 195000\ /\ 34077 = 5.7223$

$A_{c,netto} = A_c - \pi \cdot (\phi_{duct}/2)^2 = 0.9926\ m^2$

$A_{ideal} = A_c + A_p \cdot \alpha_{E,p} = 1.013\ m^2$

- Prestressing forces and stresses

The force applied to a tendon, i.e. the force at the active end during tensioning, should not exceed the following value

$P_{max} = A_p \cdot \sigma_{p,max}$

where $\sigma_{p,max} = min(0.8 \cdot f_{pk}; 0.90 \cdot f_{p0,1k})$

$P_{max} = A_p \cdot 0.80 \cdot f_{pk} = 28.5 \cdot 10^{-4} \cdot 0.80 \cdot 1770 = 4035.6\ kN$

$P_{max} = A_p \cdot 0.90 \cdot f_{p0,1k} = 28.5 \cdot 10^{-4} \cdot 0.90 \cdot 1520 = 3898.8\ kN$

$\rightarrow P_{max} = 3898.8\ kN$ and $\sigma_{p,max} = 1368\ N/mm^2$

The value of the initial prestress force at time $t = t_0$ applied to the concrete immediately after tensioning and anchoring should not exceed the following value

5.10.3 (2): Prestress force

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.3.6: Fig. 3.10

5.10.3 (2): Eq. 5.43: P_{m0} initial prestress force at time $t = t_0$

(NDP) 5.10.3 (2): $\sigma_{p,m0}(x)$ stress in the tendon immediately after tensioning or transfer

$$P_{m0}(x) = A_p \cdot \sigma_{p,m0}(x)$$

where $\sigma_{p,m0}(x) = \min\left\{0.75 f_{pk}; 0.85 f_{p0,1k}\right\}$

$$P_{m0} = A_p \cdot 0.75 \cdot f_{pk} = 28.5 \cdot 10^{-4} \cdot 0.75 \cdot 1770 = 3783.4\ kN$$

$$P_{m0} = A_p \cdot 0.85 \cdot f_{p0,1k} = 28.5 \cdot 10^{-4} \cdot 0.85 \cdot 1520 = 3682.2\ kN$$

$\rightarrow P_{m0} = 3682.2\ kN$ and $\sigma_{p,m0} = 1292\ N/mm^2$

Further calculations for the distribution of prestress forces and stresses along the beam are not in the scope of this benchmark and will not be described here. The complete diagram can be seen in benchmark 17, after the consideration of losses at anchorage and due to friction, as calculated by SOFiSTiK. There the values of $\sigma_{p,max} = 1368\ N/mm^2$ and $P_{m0} = 3682.2\ kN$ can be visualised.

In this benchmark the beam number 10010 is analysed therefore the prestressing force is obtained from TENDON:

$$P_0 = 3653\ kN$$

$$\sigma_{p,0} = 1281.755\ MPa$$

Calculating the prestressing losses due relaxation

According to DIN EN 1992-1-1, 3.3.2 (5), the value of ρ_{1000} is expressed as a percentage ratio of the initial stress and is obtained for an initial stress equal to $0.7 \cdot f_p$, where f_p is the actual tensile strength of the prestressing steel samples. For design calculations, the characteristic tensile strength (f_{pk}) is used.

$$\mu = \frac{\sigma_{p0}}{f_{pk}} = \frac{1281.755}{1770} = 0.724155 \geq \mu_{min} = 0.55$$

The formulas in section DIN EN 1992-1-1, 3.3.2 (7) are not used for the calculation, according to DiBt [3] for DIN EN 1992-1-1 [4] the relaxation values are obtained from Table 3.

Please refer to Fig. 3 to see the differences between DIN EN 1992-1-1 and EN 1992-1-1 (for t=500000 h).

Table 3: Relaxation values per hour

$\mu = R/R_n$	1	10	200	1000	5000	$5 \cdot 10^5$	$1 \cdot 10^6$
0.45	< 1 %						
0.50	-						
0.55	-					1.0	1.2
0.60	-				1.2	2.5	2.8
0.65	-			1.3	2.0	4.5	5.0
0.70	-		1.0	2.0	3.0	6.5	7.0
0.75	-	1.2	2.5	3.0	4.5	9.0	10.0

Table 3: (continued)

$\mu = R/R_n$	1	10	200	1000	5000	$5 \cdot 10^5$	$1 \cdot 10^6$
0.80	1.0	2.0	4.0	5.0	6.5	13.0	14.0

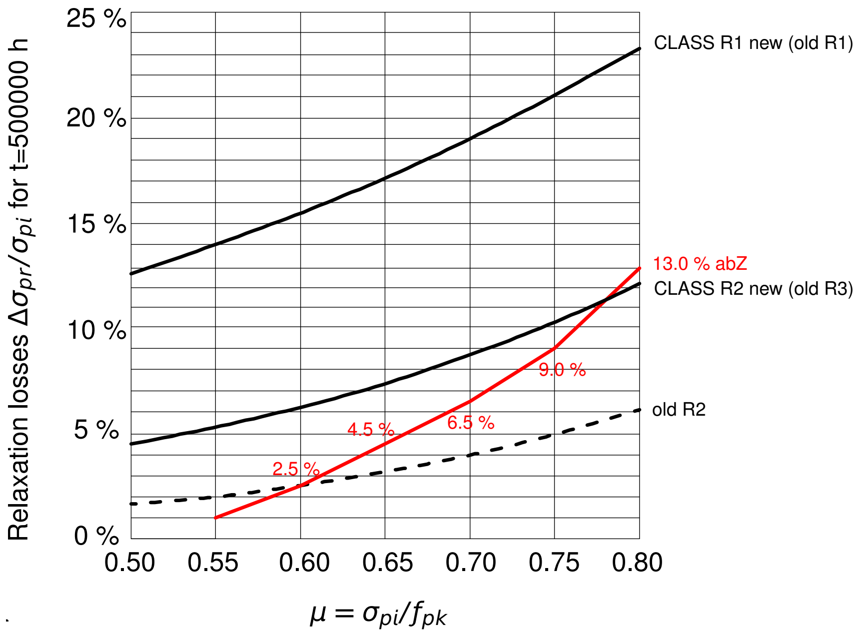

Figure 3: The relaxation loss differences according to EC2 and abZ (DiBt)

$t = 1000\ h$

AQB is interpolating the values from Table 3.

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = \frac{(3-2)\cdot(0.724-0.7)}{0.75-0.7} + 2.0 = 2.483$$

$$\Delta\sigma_{pr} = \sigma_{pi} \cdot 2.483\ \% = 1281.75 \cdot 2.483\% = 31.82\ MPa$$

AQB reduces the initial stress according to DIN 1045-1:

DIN 1045-1 8.7.3 (6)

$$\Delta P_{c+s+r} = A_p \frac{\epsilon_{cs} \cdot E_p + \Delta\sigma_{pr} + \dfrac{E_p}{E_{cm}} \cdot \phi(t,t_0) \cdot \sigma_{c,QP}}{1 + \dfrac{E_p}{E_{cm}} \cdot \dfrac{A_p}{A_c}\left(1 + \dfrac{A_c}{I_c} \cdot z_{cp}^2\right)[1 + 0.8 \cdot \phi(t,t_0)]}$$

DIN 1045-1 8.7.3 (6): ΔP_{c+s+r} Time dependent losses of prestress

Creep and shrinkage is not taken into account therefore we have:

$$\epsilon_{cs} \cdot E_p = 0$$

$$\frac{E_p}{E_{cm}} \cdot \phi(t,t_0) \cdot \sigma_{c,QP} = 0$$

$$1 + 0.8 \cdot \phi(t,t_0) = 1$$

$$\Delta\sigma_{p,c+s+r} = \frac{31.82}{1 + \frac{195000}{34077} \cdot \frac{28.5 \cdot 10^{-4}}{0.9926} \cdot \left(1 + \frac{0.9926}{0.08214} \cdot 0.3901^2\right)}$$

$$\Delta\sigma_{p,c+s+r} = \frac{31.82}{1.046644} = 30.402\ MPa$$

Now we have:

$$\sigma_{p0} = \sigma_{pg0} - 0.3 \cdot \Delta\sigma_{p,c+s+r}$$

DIN 1045-1 8.7.3 (6)

$$\sigma_{p0} = 1281.755 - 0.3 \cdot 30.4020 = 1272.631\ MPa$$

Now with the reduced stress the μ value is calculated again (iteration):

$$\mu = \frac{\sigma_{p0}}{f_{pk}} = \frac{1272.631}{1770} = 0.719 \geq \mu_{min} = 0.55$$

Interpolating the relaxation values from Table 3:

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = \frac{(3-2) \cdot (0.719 - 0.7)}{0.75 - 0.7} + 2.0 = 2.38\ \%$$

$$\Delta\sigma_{pr} = 1281.755 \cdot 2.38\% = 30.50\ MPa$$

The total loss:

$$\Delta\sigma_{pr,total} = \Delta\sigma_{pr,t} + \Delta\sigma_{pr,c}$$

$$\Delta\sigma_{pr,t} = \Delta\sigma_{pr} = 30.5093\ MPa$$

ϵ_{k0}, ϵ_{ky} and ϵ_{kz} - tendon strain

$$\Delta\sigma_{pr,c} = (\epsilon_{k0} + \epsilon_{ky} \cdot z_{cp} - \epsilon_{kz} \cdot y_{cp}) \cdot E_p$$

$$\Delta\sigma_{pr,c} = (2.457 \cdot 10^{-6} + 1.15 \cdot 10^{-5} \cdot 0.39) \cdot 1.95 \cdot 10^8/1000$$

$$\Delta\sigma_{pr,c} = 1.354\ MPa$$

$$\Delta\sigma_{pr,total} = 30.5 - 1.354 = 29.15\ MPa$$

$$\Delta P_{pr} = \Delta\sigma_{pr,total} \cdot A_p = 29.15 \cdot 28.5 \cdot 10^{-4} \cdot 1000 = 83.071\ kN$$

Prestress-force loss in [%]:

$$\Delta P_{pr}[\%] = \frac{\Delta P_{pr}}{P_0}$$

$$\Delta P_{pr}[\%] = \frac{83.08}{3653} = 0.02274 \cdot 100\% = 2.274\ \%$$

5 Conclusion

This example shows the calculation of the time dependent losses due to relaxation. It has been shown that the reference solution and the AQB solution are in very good agreement.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] DiBt. *Allgemeine bauaufsichtliche Zulassung*. Deutsches Institut für Bautechnik. 2015.

[4] *EN 1992-1-1: Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2004.