Benchmark Example No. 24

Lateral Torsional Buckling

SOFiSTiK | 2018
1 Problem Description

The problem consists of a simply supported beam with a steel I-section, which is subjected to compression and biaxial bending, as shown in Fig. 1. The beam is checked against lateral torsional buckling.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the buckling resistance of steel members. It deals with the spatial behavior of the beam and the occurrence of lateral torsional buckling as a potential mode of failure. The content of this problem is covered by the following parts of EN 1993-1-1:2005 [1]:

- Structural steel (Section 3.2)
- Classification of cross-sections (Section 5.5)
- Buckling resistance of members (Section 6.3)
- Method 1: Interaction factors $k_{ij}$ for interaction formula in 6.3.3(4) (Annex A)
3 Model and Results

The I-section, an IPE 500, with properties as defined in Table 1, is to be checked for lateral torsional buckling, with respect to EN 1993-1-1:2005 [1]. The calculation steps are presented below. The results are tabulated in Table 2 and compared to the results of reference [2].

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>S 235</td>
<td>IPE 500</td>
<td>$M_{y,Ed} = 100 , kNm$</td>
</tr>
<tr>
<td>$E = 210000 , N/mm^2$</td>
<td>$L = 3.750 , m$</td>
<td>$M_{z,Ed} = 25 , kNm$</td>
</tr>
<tr>
<td>$\gamma M_1 = 1.0$</td>
<td>$h_w = 468 , mm$</td>
<td>$q_z = 170 , kN/m$</td>
</tr>
<tr>
<td></td>
<td>$b_f = 200 , mm$</td>
<td>$N_{Ed} = 500 , kN$</td>
</tr>
<tr>
<td></td>
<td>$t_f = 16.0 , mm$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_w = 10.2 , mm$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A = 115.5 , cm^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_y = 48197 , cm^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_z = 2142 , cm^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_T = 88.57 , cm^4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_w = 1236 \times 10^3 , cm^6$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W_{pl,y} = 2194 , cm^3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W_{pl,z} = 335.9 , cm^3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W_{el,y} = 1927.9 , cm^3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W_{el,z} = 214.2 , cm^3$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>SOF.</th>
<th>Ref. [2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_y$</td>
<td>0.195</td>
<td>0.195</td>
</tr>
<tr>
<td>$\chi_y$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\lambda_z$</td>
<td>0.927</td>
<td>0.927</td>
</tr>
<tr>
<td>$\phi_z$</td>
<td>1.054</td>
<td>1.054</td>
</tr>
<tr>
<td>$\chi_z$</td>
<td>0.644</td>
<td>0.644</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$\mu_z$</td>
<td>0.937</td>
<td>0.937</td>
</tr>
</tbody>
</table>
Table 2: (continued)

<table>
<thead>
<tr>
<th></th>
<th>SOF.</th>
<th>Ref. [2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_y )</td>
<td>1.138</td>
<td>1.138</td>
</tr>
<tr>
<td>( w_z )</td>
<td>1.500</td>
<td>1.500</td>
</tr>
<tr>
<td>( C_{my,0} )</td>
<td>1.001</td>
<td>0.999</td>
</tr>
<tr>
<td>( C_{mz,0} )</td>
<td>0.771</td>
<td>0.771</td>
</tr>
<tr>
<td>( C_{my} )</td>
<td>1.001</td>
<td>1.000</td>
</tr>
<tr>
<td>( C_{mz} )</td>
<td>0.771</td>
<td>0.771</td>
</tr>
<tr>
<td>( \lambda_0 )</td>
<td>0.759</td>
<td>0.757</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>1.194</td>
<td>1.2</td>
</tr>
<tr>
<td>( C_{MLT} )</td>
<td>1.139</td>
<td>1.137</td>
</tr>
<tr>
<td>( \lambda_{LT} )</td>
<td>0.695</td>
<td>0.691</td>
</tr>
<tr>
<td>( \Phi_{LT} )</td>
<td>0.825</td>
<td>0.822</td>
</tr>
<tr>
<td>( \chi_{LT} )</td>
<td>0.787</td>
<td>0.789</td>
</tr>
<tr>
<td>( \chi_{LT,mod} )</td>
<td>0.821</td>
<td>0.826</td>
</tr>
<tr>
<td>( C_{yy} )</td>
<td>0.981</td>
<td>0.981</td>
</tr>
<tr>
<td>( C_{yz} )</td>
<td>0.862</td>
<td>0.863</td>
</tr>
<tr>
<td>( C_{zy} )</td>
<td>0.842</td>
<td>0.843</td>
</tr>
<tr>
<td>( C_{zz} )</td>
<td>1.013</td>
<td>1.014</td>
</tr>
<tr>
<td>( nm - y ) (Eq.6.61 [1])</td>
<td>0.966</td>
<td>0.964</td>
</tr>
<tr>
<td>( nm - z ) (Eq.6.62 [1])</td>
<td>0.868</td>
<td>0.870</td>
</tr>
</tbody>
</table>
4 Design Process

Design Load:

\[ M_{z,Ed} = 25 \text{kNm} \]
\[ M_{y,Ed} = -100 \text{kNm} \] at the start and end of the beam
\[ M_{y,Ed} = 199 \text{kNm} \] at the middle of the beam
\[ N_{Ed} = 25 \text{kNm} \]

The cross-section is classified as Class 1, as demonstrated in [2].

Tab. 5.5: Classification of cross-section

6.3.1.2 (1): \( N_c \) is the elastic critical force for the relevant buckling mode

\[ N_{cr,y} = \frac{\pi^2 EI_y}{L^2} = 71035 \text{kN} \]

6.3.1.2 (1): \( \lambda \) non dimensional slenderness for class 1 cross-sections

\[ \bar{\lambda}_y = \sqrt{\frac{A f_y}{N_{cr,y}}} = 0.195 \]

6.3.1.2 (4): \( \lambda \leq 0.2 \) buckling effects may be ignored

\[ \bar{\lambda}_y < 0.2 \text{ thus } \chi_y = 1.0 \]

\[ N_{cr,z} = \frac{\pi^2 EI_z}{L^2} = 3157 \text{kN} \]

\[ \bar{\lambda}_z = \sqrt{\frac{A f_y}{N_{cr,z}}} = 0.927 \]

\[ \Phi_z = 0.5 \left[ 1 + \alpha_z \left( \bar{\lambda}_z - 0.2 \right) + \bar{\lambda}_z^2 \right] \]

for rolled I-sections with \( h / b > 1.2 \) and buckling about z-z axis → buckling curve b

6.3.1.2 (2): Table 6.1: Imperfection factors for buckling curves

for buckling curve b → \( \alpha_z = 0.34 \)

\[ \Phi_z = 1.054 \]

6.3.1.2 (1): Eq. 6.49: \( \chi_z \) reduction factor for buckling

\[ \chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^2}} = 0.644 \leq 1.0 \]

The stability verification will be done according to Method 1-Annex A of EN 1993-1-1:2005 [1]. Therefore we need to identify the interaction factors according to tables A.1-A.2 of Annex A, EN 1993-1-1:2005 [1].

Auxiliary terms:

\[ \mu_y = \frac{1 - \frac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y \frac{N_{Ed}}{N_{cr,y}}} = 1.0 \]

Annex A: Tab. A.1: Interaction factors

\( k_q \) (6.3.3(4)), Auxiliary terms

1The sections mentioned in the margins refer to EN 1993-1-1:2005 [1] unless otherwise specified.
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$$\mu_z = \frac{1 - \frac{N_{Ed}}{N_{cr,z}}}{1 - \frac{N_{Ed}}{N_{Ed}}} = 0.937$$

$$w_y = \frac{W_{pl,y}}{W_{el,y}} = 1.138 \leq 1.5$$

$$w_z = \frac{W_{pl,z}}{W_{el,z}} = 1.568 > 1.5 \rightarrow w_z = 1.5$$

**Determination of \( C_{mz,0} \) factors**

The general formula for combined end moments and transverse loads is used here.

$$C_{my,0} = 1 + \left( \frac{\pi^2 EI_y}{L^2 |M_{y,Ed,right}|} - 1 \right) \frac{N_{Ed}}{N_{cr,y}}$$

$$\delta_z = 3.33 \text{ mm}$$

$$C_{my,0} = 1.001$$

The formula for linearly distributed bending moments is used here.

$$\psi_z = \frac{M_{z,ED, right}}{M_{z,Ed, left}} = 0/25 = 0$$

$$C_{mz,0} = 0.79 + 0.21\psi_z + 0.36(\psi_z - 0.33) \frac{N_{Ed}}{N_{cr,z}} = 0.771$$

$$C_{mz} = C_{mz,0} = 0.771$$

**Resistance to lateral torsional buckling**

Because \( I_T = 8.857 \times 10 - 7 m^4 < I_y = 4.820 \times 10 - 4 m^4 \), the cross-section shape is such that the member may be prone to lateral torsional buckling.

The support conditions of the member are assumed to be the so-called "fork conditions", thus \( L_{LT} = L \).

$$M_{cr,0} = \sqrt{\frac{\pi^2 EI_z}{L_{LT}^2} \left( GI_T + \frac{\pi^2 EI_w}{L_{LT}^2} \right)} = 895 kNm$$

$$\bar{\lambda}_0 = \sqrt{\frac{W_{pl,y} f_y}{M_{cr,0}}} = 0.759$$

$$N_{cr,T} = \frac{A}{I_y + I_z} \left( GI_T + \frac{\pi^2 EI_w}{L_{LT}^2} \right) = 5822 kN$$

$$C_1 = 1.194 \text{ determined by eigenvalue analysis}$$

$$\bar{\lambda}_{0lim} = 0.2 \sqrt{C_1 \left( 1 - \frac{N_{Ed}}{N_{cr,z}} \right) \left( 1 - \frac{N_{Ed}}{N_{cr,T}} \right)} = 0.205$$
Lateral torsional buckling has to be taken into account.

\[ \alpha_{LT} = 1 - \frac{I_T}{I_y} = 0.998 \geq 0 \]

Thus \( C_{LT} = 1.0 \).

The general case method is chosen here.

\[ M_{cr} = 1068 \text{ kNm}, \text{ determined by eigenvalue analysis} \]

\[ \bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} f_y}{M_{cr}}} = \sqrt{\frac{2194 \cdot 10^{-6} \cdot 235 \cdot 10^6}{1079 \cdot 10^3}} = 0.695 \]

\[ \Phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} \left( \bar{\lambda}_{LT} - 0.2 \right) + \bar{\lambda}_{LT}^2 \right] \]

for rolled I-sections and \( h / b > 2 \rightarrow \) buckling curve b

for buckling curve b \( \rightarrow \alpha_{LT} = 0.34 \)

\[ \Phi_{LT} = 0.825 \]

\[ \chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \]

\[ \chi_{LT} = 0.787 \leq 1.0 \]

\( k_c = 0.907 \) determined by eigenvalue analysis through the \( C_1 \) factor.

\[ f = 1 - 0.5 (1 - k_c) \left[ 1 - 2 \left( \bar{\lambda}_{LT} - 0.8 \right)^2 \right] = 0.959 \leq 1.0 \]

\[ \chi_{LT,mod} = \frac{\chi_{LT}}{f} = 0.821 \leq 1.0 \]

Elastic-plastic bending resistances

\[ N_{c,RK} = A \cdot f_y = 2715 \text{ kN} \]

\[ M_{pl,y,RK} = W_{pl,y} \cdot f_y = 516 \text{ kNm} \]

\[ M_{pl,z,RK} = W_{pl,z} \cdot f_y = 78.9 \text{ kNm} \]
\( \lambda_{\text{max}} = \lambda_z = 0.927 \)

\[ b_{LT} = 0.5 \cdot \alpha_{LT} \cdot \lambda_z^2 \frac{M_{y,Ed}}{\chi_{LT,mod, \gamma_1}} \frac{M_{z,Ed}}{\gamma_1} = 0.428 \]

\[ C_{yy} = 1 + (w_y - 1) \left[ 2 - \frac{1.6}{w_y} \cdot C_{my, \lambda_{\text{max}}}^2 - \frac{1.6}{w_y} \cdot C_{my, \lambda_{\text{max}}}^2 \right] \]

\[ C_{yy} = 0.981 \geq \frac{w_{el,y}}{w_{pl,y}} = 0.879 \]

\[ c_{LT} = 10 \cdot \alpha_{LT} \cdot \lambda_z^2 \frac{M_{y,Ed}}{c_{my} \cdot \chi_{LT,mod, \gamma_1}} \frac{M_{pl,y,Rk}}{\gamma_1} = 0.471 \]

\[ C_{yz} = 1 + (w_z - 1) \left[ 2 - \frac{14}{w_z} \cdot C_{my, \lambda_{\text{max}}}^2 \right] \frac{N_{Ed}}{N_{c,Rk}} \frac{c_{LT}}{\gamma_1} \]

\[ C_{yz} = 0.862 \geq 0.6 \sqrt{\frac{w_z W_{el,z}}{w_y W_{pl,z}}} = 0.439 \]

\[ d_{LT} = 2 \cdot \alpha_{LT} \cdot \lambda_z^2 \frac{M_{y,Ed}}{0.1 + \lambda_z} \frac{M_{z,Ed}}{c_{my} \cdot \chi_{LT,mod, \gamma_1}} \frac{M_{pl,z,Rk}}{\gamma_1} \]

\[ = 0.348 \]

\[ C_{zy} = 1 + (w_y - 1) \left[ 2 - \frac{14}{w_y} \cdot C_{my, \lambda_{\text{max}}}^2 \right] \frac{N_{Ed}}{N_{c,Rk}} \frac{d_{LT}}{\gamma_1} \]

\[ C_{zy} = 0.842 \geq 0.6 \sqrt{\frac{w_y W_{el,y}}{w_z W_{pl,z}}} = 0.459 \]

\[ e_{LT} = 1.7 \cdot \alpha_{LT} \cdot \lambda_z^2 \frac{M_{y,Ed}}{0.1 + \lambda_z} \frac{M_{pl,y,Rk}}{c_{my} \cdot \chi_{LT,mod, \gamma_1}} \]

\[ = 0.721 \]
\[ C_{zz} = 1 + (w_z - 1) \left[ 2 \frac{1.6}{w_z} \cdot c_{mz}^2 \lambda_{max} \right. - \frac{1.6}{w_z} \cdot c_{mz}^2 \lambda_{max}^2 \left. - e_{LT} \right] \cdot \frac{N_{Ed}}{N_{c,Rk}} \frac{c_{mz}}{\gamma_{M1}} \]

\[ C_{zz} = 1.013 \geq \frac{W_{el,z}}{W_{pl,z}} = 0.667 \]
Verification

According to 1993-1-1:2005, 6.3.3 (4), members which are subjected to combined bending and axial compression should satisfy:

\[
\frac{N_{Ed}}{N_{c,Rk}} + \mu_y \left[ \frac{C_{mL}}{X_{LT,mod}} \left( \frac{1 - \frac{N_{Ed}}{N_{cr,y}}}{C_{yy}} \cdot \frac{M_{pL,Rk}}{\gamma M_1} \right) \right] + 0.6 \sqrt{\frac{w_z}{w_y} \frac{C_{mz} M_{z,Ed}}{C_{yz} M_{pL,Rk}} \left( 1 - \frac{N_{Ed}}{N_{cr,z}} \right) \frac{M_{pL,Rk}}{\gamma M_1}}
\]

6.3.3 (4): Eq. 6.61: Members which are subjected to combined bending and axial compression should satisfy:

\[
\frac{500}{2715} + 1.0 \left[ \frac{1.139}{0.821} \frac{1.001 \cdot 198.9}{1.0} \right] + 0.6 \sqrt{1 - \frac{500}{3157}} \frac{0.771 \cdot 25}{1.0} = 0.966 \leq 1.00
\]

→ Satisfactory

\[
\frac{N_{Ed}}{N_{c,Rk}} + \mu_z \left[ 0.6 \sqrt{\frac{w_y}{w_z} \frac{C_{mL}}{X_{LT,mod}} \left( 1 - \frac{N_{Ed}}{N_{cr,y}} \right) \frac{M_{pL,Rk}}{\gamma M_1} \right] + \frac{C_{mz} M_{z,Ed}}{\left( 1 - \frac{N_{Ed}}{N_{cr,z}} \right) \frac{M_{pL,Rk}}{\gamma M_1}}
\]

6.3.3 (4): Eq. 6.62: Members which are subjected to combined bending and axial compression should satisfy:

\[
\frac{500}{2715} + 0.937 \left[ +0.6 \sqrt{\frac{1.138}{1.500} \frac{1.139}{0.821}} \right] = 0.964 + \frac{1.001 \cdot 198.9}{1.0} + 0.771 \cdot 25
\]
\[= 0.868 \leq 1\]

→ Satisfactory
5 Conclusion

This example shows the check for lateral torsional buckling of steel members. The small deviations that occur in some results come from the fact that there are some small differences in the sectional values between SOFiSTiK and the reference solution. Therefore, these deviations are of no interest for the specific verification process. In conclusion, it has been shown that the results are reproduced with excellent accuracy.

6 Literature
