Benchmark Example No. 8

Design of a Rectangular CS for Shear and Axial Force
This manual is protected by copyright laws. No part of it may be translated, copied or reproduced, in any form or by any means, without written permission from SOFiSTiK AG. SOFiSTiK reserves the right to modify or to release new editions of this manual.

The manual and the program have been thoroughly checked for errors. However, SOFiSTiK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
1 Problem Description

The problem consists of a rectangular section, symmetrically reinforced for bending, as shown in Fig. 1. The cross-section is designed for a shear force $V_{Ed}$ and a compressive force $N_{Ed}$ and and the required reinforcement is determined.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to shear force and axial force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1] [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)
- Reinforcement (Section 9.2.2)

![Figure 2: Shear Reinforced Members](image)

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and defined in DIN EN 1992-1-1:2004 [1].
3 Model and Results

The rectangular cross-section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], under a shear force of $343.25 \, kN$ and a compressive axial force of $500.0 \, kN$. The reference calculation steps are presented below and the results are given in Table 2.

### Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C , 30/37$</td>
<td>$h = 50.0 , cm$</td>
<td>$V_{Ed} = 343.25 , kN$</td>
</tr>
<tr>
<td>$B , 500A$</td>
<td>$b = 30 , cm$</td>
<td>$N_{Ed} = 500.0 , kN$</td>
</tr>
<tr>
<td></td>
<td>$d = 45.0 , cm$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_{s,\text{tot}} = 38.67 , cm^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_{V,\text{l}} = 3.6 , cm$</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{sw} , s , [cm^2/m]$</td>
<td>11.27</td>
<td>11.27</td>
</tr>
<tr>
<td>$V_{Rd,c} , [kN]$</td>
<td>132.71</td>
<td>132.71</td>
</tr>
<tr>
<td>$\cot \theta$</td>
<td>1.82</td>
<td>1.82</td>
</tr>
</tbody>
</table>
4 Design Process\(^1\)

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:\(^2\)

Material:

Concrete: \(\gamma_c = 1.50\)

Steel: \(\gamma_s = 1.15\)

\[f_{ck} = 30 \text{ MPa}\]
\[f_{cd} = a_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 30 / 1.5 = 17.0 \text{ MPa}\]

\[f_{yk} = 500 \text{ MPa}\]
\[f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78 \text{ MPa}\]

Design Load: \(V_{Ed} = 343.25 \text{kN}\)

\(N_{Ed} = -500.0 \text{kN}\)

\[z = \max \{d - c_{V,l} - 30 \text{mm}; d - 2 \cdot c_{V,l}\}\]

\[z = \max \{384; 378\} = 384 \text{mm}\]

\[V_{Rd,c} = \left[ C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_{cp} \right] \cdot b_w \cdot d\]

with a minimum of

\[(V_{\min} + 0.12 \cdot \sigma_{cp}) \cdot b_w \cdot d\]

\[C_{Rd,c} = 0.15 / \gamma_c = 0.1\]

\[k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{450}} = 1.6667 < 2.0\]

\[\rho_1 = \frac{A_s,\text{tot}}{b_w d} = 0.01432 < 0.02\]

\[V_{Rd,c,\min} = (V_{\min} + 0.12 \cdot \sigma_{cp}) \cdot b_w \cdot d\]

\[V_{\min} = (0.0525 / \gamma_c) \cdot k^{3/2} \cdot f_{ck}^{1/2} = 0.41249\]

\[V_{Rd,c,\min} = 109.68 \text{kN}\]

\[\sigma_{cp} = N_{Ed} / A_c < 0.2 \cdot f_{cd}\]

\[\sigma_{cp} = -500 \cdot 10^{-3} / 0.15 \cdot 10^6 = -3.3333 \text{ N/mm}^2 < 3.4\]

\[V_{Rd,c} = \left[ 0.1 \cdot 1.6667 \cdot (1.432 \cdot 30)^{1/3} + 0.112 \cdot 3.3333 \right] \cdot 0.3 \cdot 0.45 = 132.71 \text{kN}\]

\[V_{Ed} > V_{Rd,c} \rightarrow \text{shear reinforcement is required}\]

\(^1\)The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7(2), Fig. 3.8, which can be seen in Fig. 3.

\(^2\)The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.
Design of a Rectangular CS for Shear and Axial Force

1.0 ≤ cot θ ≤ \( \frac{1.2 + 1.4 \sigma_{cd} / f_{cd}}{1 - V_{Rd,cc} / V_{Ed}} \) ≤ 3.0

\( V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left( 1 - 1.2 \frac{\sigma_{cd}}{f_{cd}} \right) \cdot b_w \cdot z \)

\( \sigma_{cp} = \frac{N_{Ed}}{A_c} \)

\( \sigma_{cd} = -500 \cdot 10^{-3} / 0.15 \cdot 10^6 = -3.3333 \text{ N/mm}^2 \)

\( V_{Rd,cc} = 0.5 \cdot 0.48 \cdot 30^{1/3} \cdot \left( 1 - 1.2 \frac{3.3333}{17.0} \right) \cdot 0.3 \cdot 0.384 \)

\( V_{Rd,cc} = 65.6948 \text{ kN} \)

6.7DE: \( \sigma_{cd} > 0 \) for compression

6.2.3 (2): The angle \( \theta \) should be limited by Eq. 6.7DE

6.2.3 (3): Eq. 6.8

\( f_{ywd} = f_{yk} / Y_k = 435 \text{ MPa} \)

6.2.3 (3): Eq. 6.9

Maximum shear force \( V_{Rd,max} \) occurs for \( \theta = 45^\circ \): cot \( \theta = \tan \theta = 1 \)

\( V_{Rd,max} = b_w \cdot z \cdot \nu_1 \cdot f_{cd} / (\cot \theta + \tan \theta) \)

\( V_{Rd,max} = 0.3 \cdot 0.384 \cdot 0.75 \cdot 17 / (1 + 1) = 734.4 \text{ kN} \)
5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular beam cross-section under shear with compressive axial force. It has been shown that the results are reproduced with excellent accuracy.

6 Literature
