Benchmark Example No. 9

Design of a Rectangular CS for Shear and Torsion

SOFiSTiK | 2018
1 Problem Description

The problem consists of a rectangular section, symmetrically reinforced for bending, as shown in Fig. 1. The cross-section is designed for shear force $V_{Ed}$ and torsion $T_{Ed}$ and the required shear and torsion reinforcement is determined.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to shear force and torsion. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1] [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear (Section 6.2) and torsion design (Section 6.3)
- Reinforcement (Section 9.2.2, 9.2.3)

![Figure 2: Torsion Reinforced Members](image)

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).
3 Model and Results

The rectangular cross-section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], under shear force of 175.0 kN and torsional moment 35 kNm. The reference calculation steps [3] are presented below and the results are given in Table 2.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 35/45</td>
<td>$h = 70.0 , cm$</td>
<td>$V_{Ed} = 175.0 , kN$</td>
</tr>
<tr>
<td>B 500A</td>
<td>$b = 30 , cm$</td>
<td>$T_{Ed} = 35.0 , kNm$</td>
</tr>
<tr>
<td></td>
<td>$d = 65.0 , cm$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_{s, tot} = 26.8 , cm^2$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>SOF</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{sw} / s_w (T) , [cm^2/m]$</td>
<td>3.35</td>
<td>3.35</td>
</tr>
<tr>
<td>$A_{sl} (T) , [cm^2]$</td>
<td>5.37</td>
<td>5.37</td>
</tr>
<tr>
<td>$A_{sw, total} / s , [cm^2/m]$</td>
<td>13.60</td>
<td>13.59</td>
</tr>
<tr>
<td>$V_{Rd, max} , [kN]$</td>
<td>1303.05</td>
<td>1303.03</td>
</tr>
<tr>
<td>$T_{Rd, max} , [kNm]$</td>
<td>124.95</td>
<td>124.95</td>
</tr>
</tbody>
</table>
4 Design Process

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

$f_{ck} = 30$ MPa

$f_{cd} = a_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 35 / 1.5 = 19.833$ MPa

$f_{yk} = 500$ MPa

$f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78$ MPa

Design Load:

$V_{Ed} = 175.0$ kN, $T_{Ed} = 35.0$ kN

$T_{Ed} \leq \frac{V_{Ed} \cdot b_w}{4.5}$

$35 > \frac{175 \cdot 0.3}{4.5} = 11.66$

→ Eq. NA.6.31.1 is not fulfilled

$V_{Ed} \left[ 1 + \frac{4.5 \cdot T_{Ed}}{V_{Ed} \cdot b_w} \right] \leq V_{Rd,c}$

$V_{Rd,c} = \left[ C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_y \right] \cdot b_w \cdot d$

$C_{Rd,c} = 0.15 / \gamma_c = 0.1$

$k = 1 + \sqrt{\frac{200}{d}} = 1.5773 < 2.0$

$\rho_1 = \frac{A_{sl}}{b_w \cdot d}$

$\rho_1 = \frac{26.8}{0.3 \cdot 65} = 0.0137 < 0.02$

$V_{Rd,c} = \left[ 0.1 \cdot 1.5773 \cdot (100 \cdot 0.0137 \cdot 35)^{1/3} + 0 \right] \cdot 0.3 \cdot 0.65$

$V_{Rd,c} = 111.74$ kN

$175 \cdot \left[ 1 + \frac{4.5 \cdot 35}{175 \cdot 0.3} \right] = 700 > 111.74$

→ requirement of Eq. NA.6.31.2 is not met

\(^1\) The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7(2), Fig. 3.8, which can be seen in Fig. 3.

\(^2\) The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.
**Torsional reinforcement**

\[ t_{eff,1} = t_{eff,2} = 2 \cdot 50 = 100 \text{ mm} \quad (s_o = s_u = s_s = 50 \text{ mm}) \]

\[ A_k = (h - s_u - s_o) \cdot (b_w - t_{eff,1}) = 100 \text{ mm} \]

\[ A_k = (700 - 50 - 50) \cdot (300 - 100) = 120000 \text{ mm}^2 = 0.12 \text{ m}^2 \]

\[ u_k = 2 \cdot [(700 - 50 - 50) + (300 - 100)] = 1600 \text{ mm} = 1.6 \text{ m} \]

Simplifying, the reinforcement for torsion may be determined alone under the assumption of \( \cot \theta = 1.0 \), \( \theta = 45^\circ \) and be added to the independently calculated shear force reinforcement.

\[ A_{sw, requ} / s_w = T_{Ed} \cdot \tan \theta / (f_{yd} \cdot 2A_k) \]

\[ A_{sw, requ} / s_w = 350 \cdot 1.0 / (435 \cdot 2 \cdot 0.12) = 3.35 \text{ cm}^2/\text{m} \text{ (T)} \]

\[ A_{sl, requ} = T_{Ed} \cdot u_k \cdot \cot \theta / (f_{yd} \cdot 2A_k) = 5.37 \text{ cm}^2 \text{ (T)} \]

**Torsional resistance moment**

\[ T_{Rd, max} = 2 \cdot \nu \cdot f_{cd} \cdot A_k \cdot t_{eff,i} \cdot \sin \theta \cdot \cos \theta \]

\( \sin \theta \cdot \cos \theta = 0.5 \text{ since } \theta = 45^\circ \)

\[ T_{Rd, max} = 124.95 \text{ kNm} \]

**Check of the concrete compressive strut bearing capacity for the load combination of shear force and torsion**

The maximum resistance of a member subjected to torsion and shear is limited by the capacity of the concrete struts. The following condition should be satisfied:

\[
\left[ \frac{T_{Ed}}{T_{Rd, max}} \right]^2 + \left[ \frac{V_{Ed}}{V_{Rd, max}} \right]^2 \leq 1.0
\]

For the T+V utilization, SOFiSTiK uses:

\[
\sqrt{\left[ \frac{T_{Ed}}{T_{Rd, max}} \right]^2 + \left[ \frac{V_{Ed}}{V_{Rd, max}} \right]^2} \leq 1.0
\]

\[ z = \max \{d - c_{V,1} - 30 \text{ mm}; d - 2 \cdot c_{V,1}\} \]

\[ c_{V,1} = s_o - D_o/2 = 50 - 28/2 = 36 \text{ mm} \]

\[ z = \max \{584; 578\} = 584 \text{ mm} \]

\[ V_{Rd, max} = b_w \cdot z \cdot z_1 \cdot f_{cd} / (\cot \theta + \tan \theta) \]

\[ V_{Rd, max} = 0.3 \cdot 0.584 \cdot 0.75 \cdot 19.833 / (1 + 1) = 1303.03 \text{ kN} \]

\[
\left[ \frac{35}{124.95} \right]^2 + \left[ \frac{175}{1303.04} \right]^2 = 0.0965 < 1
\]

**Shear reinforcement**

\[ A_{sw, requ} / s = V_{Ed} / (f_{y wd} \cdot z \cdot \cot \theta) = 6.89 \text{ cm}^2/\text{m} \]
Total required reinforcement

Required torsional reinforcement:

\[ 2 \cdot A_{sw} / s = 2 \cdot 3.35 = 6.7 \text{ cm}^2 / m \text{ (double-shear connection)} \]

Total reinforcement: \( A_{sw,\text{total}} / s = 6.7 + 6.89 = 13.59 \text{ cm}^2 / m \)

Check the maximum allowable compressive stress

\[ \sigma_{II} < \sigma_{c,v+t} \]

\(-4.91 \text{ MPa} < -14.88 \text{ MPa} \rightarrow \text{OK} \)
5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular beam cross-section under shear and torsion. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

