Benchmark Example No. 2

Creep and Shrinkage Calculation using the Model Code 1990
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The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
Overview

| Design Code Family(s): MC | Design Code(s): MC 1990 | Module(s): AQB, CSM | Input file(s): creep_shrinkage_mc90.dat |

1 Problem Description

The problem consists of a simply supported beam with a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The total creep and shrinkage is calculated.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the calculation of creep and shrinkage on a prestressed concrete section, subject to bending and prestress force. The content of this problem is covered by the following parts of CEB-FIP Model Code 1990 [1]:

- Creep and Shrinkage (Section 2.1.6.4)
- Temperature effects (Section 2.1.8)

In this Benchmark the total creep and shrinkage will be examined.

3 Model and Results

Benchmark 17 is here extended for the case of creep and shrinkage developing on a prestressed concrete simply supported beam. The analysed system can be seen in Fig. 2, with properties as defined in Table 1. Further information about the tendon geometry and prestressing can be found in Benchmark 17. The beam consists of a rectangular section and is prestressed and loaded with its own weight. A calculation of the creep and shrinkage is performed with respect to CEB-FIP Model Code 1990 [1].

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 35/45</td>
<td>$h = 100.0 \text{ cm}$</td>
<td>$t_0 = 28 \text{ days}$</td>
</tr>
<tr>
<td>Y 1770</td>
<td>$b = 100.0 \text{ cm}$</td>
<td>$t_s = 0 \text{ days}$</td>
</tr>
</tbody>
</table>
Table 1: (continued)

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RH = 80%$</td>
<td>$L = 20.0\ m$</td>
<td>$t = 36500\ days$</td>
</tr>
<tr>
<td></td>
<td>$A_p = 28.5\ cm^2$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Simply Supported Beam

Table 2: Results

<table>
<thead>
<tr>
<th>Result</th>
<th>AQB</th>
<th>CSM+AQB</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{cs}$</td>
<td>$-25.1 \cdot 10^{-5}$</td>
<td>$25.1 \cdot 10^{-5}$</td>
<td>$25.146 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>1.57</td>
<td>-</td>
<td>1.566</td>
</tr>
<tr>
<td>$\phi(t, t_0)$</td>
<td>1.48</td>
<td>1.476</td>
<td>1.47</td>
</tr>
</tbody>
</table>

**Note:** The results from SOFiSTiK are rounded for output.
## 4 Design Process

Design with respect to CEB-FIP Model Code 1990 [1]

Material:

Concrete: C 35/45

\[ E_{cm} = 35000 \text{ N/mm}^2 \]

\[ f_{ck} = 35 \text{ N/mm}^2 \]

\[ f_{cm} = 43 \text{ N/mm}^2 \]

Prestressing Steel: Y 1770

\[ E_p = 195000 \text{ N/mm}^2 \]

\[ f_{pk} = 1770 \text{ N/mm}^2 \]

### CALCULATION OF TOTAL SHRINKAGE AND SWELLING at \( x = 10.0 \text{ m midspan} \):

\[ t_0 = 28 \text{ days} \]

\[ t_s = 0 \text{ days} \]

\[ t = 365000 \text{ days} \]

The total shrinkage or swelling strain \( \varepsilon_{cs}(t, t_s) \) may be calculated from

\[ \varepsilon_{cs}(t, t_s) = \varepsilon_{cs0} \cdot \beta_s(t - t_s) \]

**Calculating the notional shrinkage:**

The notional shrinkage coefficient may be obtained from

\[ \varepsilon_{cs0} = \varepsilon_s(f_{cm}) \cdot \beta_{RH} \]

with:

\[ \varepsilon_s(f_{cm}) = \left[ 160 + 10 \cdot \beta_{sc} \cdot \left( 9 - \frac{f_{cm}}{f_{cm0}} \right) \right] \cdot 10^{-6} \]

\[ \varepsilon_s(f_{cm}) = \left[ 160 + 10 \cdot 5 \cdot \left( 9 - \frac{43}{10} \right) \right] \cdot 10^{-6} \]

\[ \varepsilon_s(f_{cm}) = 39.5 \cdot 10^{-5} \]

\[ \beta_{RH} = -1.55 \cdot \beta_{sRH} \text{ for } 40 \% \leq RH < 99 \% \]

\[ \beta_{sRH} = 1 - \left( \frac{RH}{RH_0} \right)^3 = 1 - \left( \frac{80}{100} \right)^3 = 0.488 \]

\[ \beta_{RH} = -1.55 \cdot 0.488 = -0.7564 \]

\[ \varepsilon_{cs0} = 39.5 \cdot 10^{-5} \cdot (-0.7564) = -29.8778 \]
The development of shrinkage with time is given by:

\[ \beta_s(t - t_s) = \left[ \frac{(t - t_s)/t_1}{350 \cdot (h/h_0)^2 + (t - t_s)/t_1} \right]^{0.5} \]

SOFISTiK accounts not only for the age at start of drying \( t_s \) but also for the influence of the age of prestressing, so the time development function reads:

\[ \beta_s = \beta_s(t - t_s) - \beta_s(t_0 - t_s) \]

\[ \beta_s = \left[ \frac{36500}{350 \cdot 5^2 + 36500} \right]^{0.5} \left[ \frac{28}{350 \cdot 5^2 + 28} \right]^{0.5} \]

\[ \beta_s = 0.8981 - 0.05647 = 0.8416 \]

The total shrinkage or swelling strain is calculated:

\[ \varepsilon_{cs}(t, t_s) = \varepsilon_{cs0} \cdot \beta_s \]

\[ \varepsilon_{cs}(t, t_s) = -29.8778 \cdot 10^{-5} \cdot 0.8416 = 25.146 \cdot 10^{-5} \]

**CALCULATION OF TOTAL CREEP** at \( x=10.0 \) m midspan:

The creep coefficient may be calculated from:

\[ \phi(t, t_0) = \phi_0 \cdot \beta_c(t - t_0) \]

The notional creep coefficient may be estimated from:

\[ \phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \]

with:

\[ \phi_{RH} = 1 + \frac{1 - (RH/RH_0)}{0.46 \cdot (h/h_0)^{1/3}} \]

\[ \phi_{RH} = 1 + \frac{1 - (80/100)}{0.46 \cdot (500/100)^{1/3}} = 1 + \frac{0.2}{0.78658} = 1.254 \]

\[ \beta(f_{cm}) = \left( \frac{f_{cm}}{f_{cm0}} \right)^{0.5} = \left( \frac{5.3}{43/10} \right)^{0.5} = 2.556 \]

The adjusted time \( t_0 \) is given by:

\[ t_{0,T} = \sum_{i=1}^{n} \Delta t_i \cdot \exp \left[ 13.65 - \frac{4000}{273 + T(Dt_i/T_0)} \right] \]

\[ t_{0,T} = \sum_{i=1}^{n} 28 \cdot \exp \left[ 13.65 - \frac{4000}{273 + 20/1} \right] = 27.947 \text{ days} \]

\[ t_{0,adj} = t_{0,T} \cdot \left[ \frac{9}{2 + (t_{0,T}/t_{1,T})^{1.2}} + 1 \right] \geq 0.5 \text{ days} \]

\[ t_{0,adj} = 27.947 \cdot \left[ \frac{9}{2 + 27.947^{1.2}} + 1 \right]^{0} = 27.947 \geq 0.5 \text{ days} \]

\[ \beta(t_0) = \frac{1}{0.1 + (t_0/t_1)^{0.2}} = \frac{1}{0.1 + (27.947/1)^{0.2}} = 0.48862 \]
The development of creep with time is given by:

\[ \beta_c(t - t_0) = \left[ \frac{(t - t_0)/t_1}{\beta_h + (t - t_0)/t_1} \right]^{0.3} \]

2.1.6.4.3(b): Eq. 2.1-70

with:

\[ \beta_h = 150 \cdot \left( 1 + \left( 1.2 \cdot \frac{RH}{RH_0} \right)^{18} \right) \cdot \frac{h}{h_0} + 250 \leq 1500 \]

2.1.6.4.3(b): Eq. 2.1-71; \( t_1 = 1 \) day; \( RH_0 = 100 \% \); \( h_0 = 100 \text{ mm} \)

\[ \beta_H = 1359.702 \leq 1500 \]

\[ \beta_c(t - t_0) = \left[ \frac{(36500 - 28)/1}{1359.702 + (36500 - 28)/1} \right]^{0.3} = 0.989 \]

\[ \phi_0 = 1.254 \cdot 2.556 \cdot 0.48862 = 1.566 \]

The creep coefficient:

\[ \phi(t, t_0) = 1.56613 \cdot 0.989 = 1.5489 \]

The creep value is related to the tangent Youngs modulus, where the tangent modulus being defined as \( 1.05 \cdot E_{cm} \). To account for this, SOFiSTiK adopts this scaling for the computed creep coefficient (in SOFiSTiK, all computations are consistently based on the secant modulus of elasticity).

\[ \phi(t, t_0) = \frac{1.5489}{1.05} = 1.47 \]
5 Conclusion

This example shows the calculation of the creep and shrinkage using Model Code 1990 [1]. It has been shown that the results are in very good agreement with the reference solution.

6 Literature