Benchmark Example No. 1

Punching of flat slabs acc. SIA 262

SOFiSTiK | 2018

VERiFiCATiON MANUAL
DCE-SIA1: Punching of flat slabs acc. SIA 262

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Front Cover

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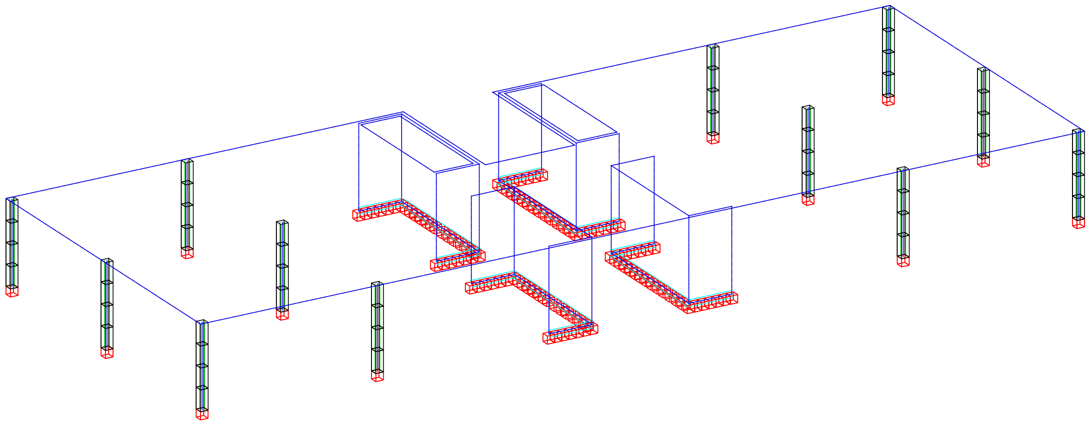
Overview	
Design Code Family(s):	SIA
Design Code(s):	SIA 262
Module(s):	BEMESS
Input file(s):	slab_punching_sia262_2013.dat

1 Problem Description

The problem consists of a flat slab. The structure under consideration is a five-storey residential building with the geometry and main dimensions given in Fig. 1. The design of slab against punching at the columns is discussed in the following.

For the concrete, strength class C30/37 ($f_{ck} = 30\ MPa$, $\gamma_c = 1.5$) is assumed, for the reinforcing steel, grade B500B ($f_{yk} = 500\ MPa$, $E_s = 205\ GPa$, $\gamma_s = 1.15$, ductility class B). The factored design load accounting for self-weight, dead load and imposed load is $q_d = 15.6 kN/m^2$.



(a) SOFiSTiK Model

Figure 1: View of building

The width of the slab is $h = 26\ cm$ as shown in Fig. 2.

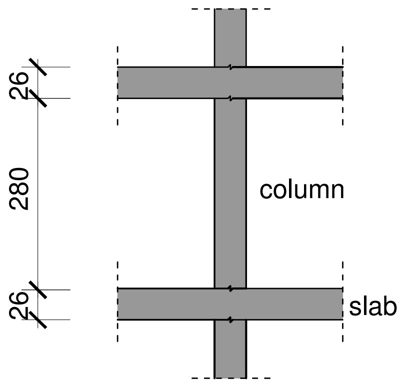


Figure 2: Section through flat slab and supporting columns

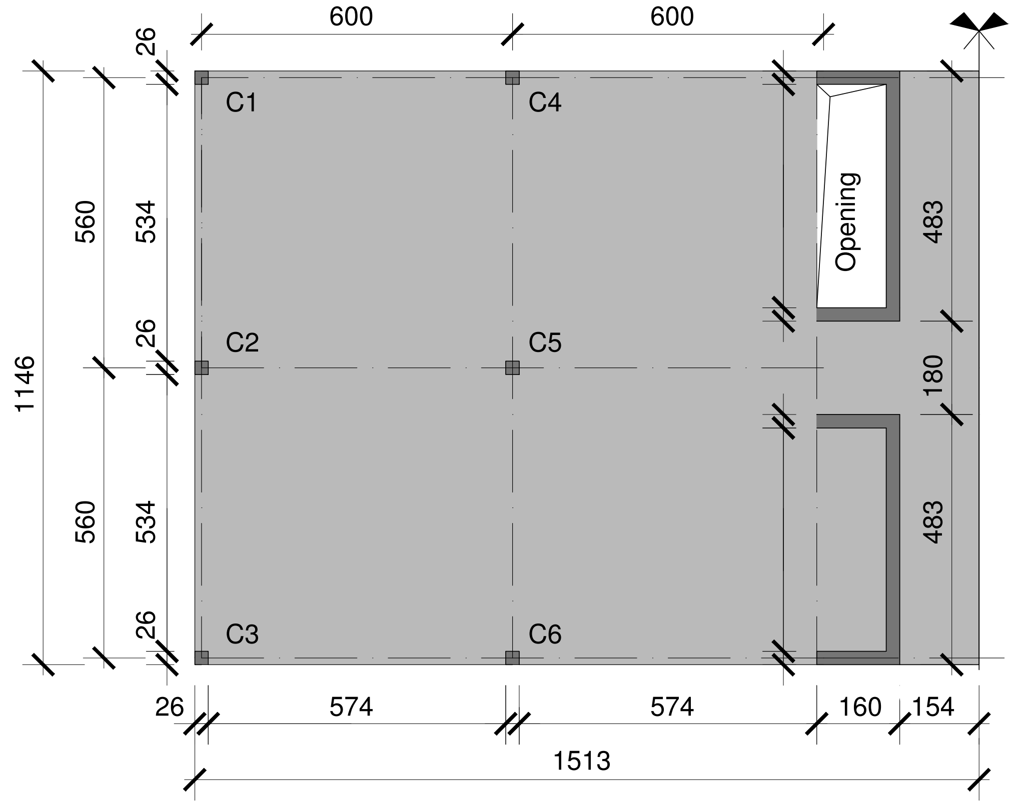


Figure 3: Main dimensions in [cm], $l_x = 6.00\ m$, $l_y = 5.60\ m$

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C30/37$	$h = 26\ cm$	$q_d = 15.6 kN/m^2$
$B500B$	$d_1 = 4.0\ cm$	
	$d = 21.0\ cm$	

2 Reference Solution

This example is concerned with the punching of flat slabs. The content of this problem is covered by the following parts of SIA 262:2013 [1]:

- Construction materials (Section 2.2)
- Dimensoning values (Section 4.2)
- Shear force (Section 4.3.3)
- Punching (Section 4.3.6)

3 Model and Results

The goal of the preliminary design is to check if the dimensions of the structure are reasonable with respect to the punching shear strength and if punching shear reinforcement is needed.

In the reference example the reaction forces are estimated by using contributive areas. The results are given in Table 2.

Table 2: Results

Result	SOF.	Ref.
Inner column C5 (Node 1070)		
u	$1.681\ m$	$1.681\ m$
k_e	0.9	0.9
u_{red}	$1.5129\ m$	$1.5129\ m$
ψ_x	$1.31\ \%$	$1.325\ \%$
ψ_y	$1.41\ \%$	$1.424\ \%$
k_r	1.03	1.02786
$V_{Rd,c}$	$349.2\ kN$	$347.36\ kN$
$V_{Rdc,max}$	$698.4\ kN$	$694.69\ kN$
A_{sw}	$8.79\ cm^2$	$8.766\ cm^2$

4 Design Process[1]

The calculation steps of the reference solution are presented below.

STEP 1: Material

SIA 262:2013; 4.2.1

Concrete 30/37

Characteristic value of cylinder compressive strength

$$f_{ck} = 30\ N/mm^2$$

Conversion factor η_{fc}; 4.2.1.2; Eq. (26)

$$\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} = \left(\frac{30}{30}\right)^{1/3} = 1.00 \leq 1.00$$

The dimensioning value of the concrete compressive strength; 2.3.2.3; Eq. (2)

$$f_{cd} = \frac{\eta_{fc} \cdot \eta_t \cdot f_{ck}}{\gamma_c} = \frac{1 \cdot 1 \cdot 30}{1.5} = 20\ N/mm^2$$

The dimensioning value of shear stress limit; 2.3.2.4; Eq.(3)

$$\tau_{cd} = \frac{0.3 \cdot \eta_t \cdot \sqrt{(f_{ck})}}{\gamma_c} = \frac{0.3 \cdot 1.0 \cdot \sqrt{30}}{1.5} = 1.095\ N/mm^2$$

$$D_{max} = 32mm$$

STEP 2: Reinforcement

SIA 262:2013; 4.2.2

Steel B500B (flexural and transverse reinforcement)

$$f_{yd} = 435\ MPa$$

$$E_s = 205000\ MPa$$

Ductility class: B

For $\phi 10$ @200 $mm/\phi 16$ @200 mm

STEP 3: Cross-section

$$\begin{aligned} d &= h - d_1 \\ &= 26 - \left(4.8 + \frac{1.6}{2}\right) \\ &= 20.4\ cm \end{aligned}$$

4.3.6.2; Fig. 21 and Fig. 22

STEP 4: Calculating the control perimeter

Inner:

$$\begin{aligned} u &= 2 \cdot a + 2 \cdot b + d_v \cdot \pi \\ &= 2 \cdot 26 + 2 \cdot 26 + 20.4 \cdot 3.14 \\ &= 168.1\ cm \end{aligned}$$

In BEMESS the shear force V_d is equal to column reaction force minus the applied load within the control perimeter $g_d \cdot A_c$. The value q_d is not taken into account.

$$V_d = 686.1\ kN$$

[1] The sections mentioned in the margins refer to SIA 262:2013 [1] unless otherwise specified.

Bemess takes into account the min. value of k_e.

For inner columns $k_e = 0.9$, SIA 262:2013, 4.3.6.2.5

$$k_e = min\left(0.9; 1 + \frac{1}{1 + \frac{e_u}{b}}\right) \tag{1}$$

$$e_u = \left|\frac{M_d}{V_d} - \Delta e\right| = \left|\frac{5.46\ kNm}{686.1\ kN}\right| = 7.95\ mm$$

In case of inner columns, the centroid of the column corresponds to the centroid of the control perimeter. Therefore, $\Delta e = 0$

$$\begin{aligned}A_c &= b_c^2 + 4 \cdot b_c \cdot \frac{d_v}{2} + \frac{d_v^2}{4} \cdot \pi \\ &= 0.26^2 + 4 \cdot 0.26 \cdot \frac{0.204}{2} + \frac{0.204^2}{4} \cdot \pi \\ &= 0.2063\ m^2\end{aligned}$$

$$b_u = \sqrt{\frac{4}{\pi} \cdot A_c} = \sqrt{\frac{4}{\pi} \cdot 0.2063} = 0.5129\ m = 512.9\ mm$$

According to Eq. 1:

$$\begin{aligned}k_e &= min\left(0.9, \frac{1}{1 + \frac{e_u}{b}}\right) \\ &= min\left(0.9, \frac{1}{1 + \frac{e_u}{b}}\right) \\ &= min\left(0.9, \frac{1}{1 + \frac{7.95}{512.9}}\right) \\ &= min(0.9, 0.98) \\ &= 0.9\end{aligned}$$

Where e_u is the eccentricity of the resultant of shear forces with respect to the centroid of the basic control perimeter and b_u is the diameter of a circle with the same surface as the region inside the basic control perimeter.

Reduced control perimeter is calculated:
$u_{red} = k_e \cdot u = 0.9 \cdot 168.1 = 151.29\ cm$

STEP 5: Rotations

The distances $r_{s,x}$ and $r_{s,y}$ are calculated from the results of the flexural analysis, one can obtain the distances between the center of the column and the point, at which the bending moments are zero.

$$r_{sx} = 1.166\ m$$

$$r_{sy} = 1.248\ m$$

The average moment of the strip is calculated by the integration of the moments at the strip section. Since the flexural moments $m_{d,x}$ and $m_{d,y}$ are negative, the absolute value of the twisting moment $m_{d,x}$ needs to be subtracted so that the absolute value of $m_{sd,x}$ and $m_{sd,y}$ will be maximized:

$$m_{sd,x} = m_{d,x} - |m_{d,xy}|$$

$$m_{sd,y} = m_{d,y} - |m_{d,xy}|$$

By using FEM analysis:

$$m_{sd,x} = 105.53\ kNm/m$$

$$m_{sd,y} = 105.81\ kNm/m$$

The representative width b_s; 4.3.6.4.6; Eq. (60)

The representative width is calculated:

$$\begin{aligned} b_s &= 1.5 \cdot \sqrt{r_{sx} \cdot r_{sy}} \leq l_{min} \\ &= 1.5 \cdot \sqrt{1.248 \cdot 1.166} \\ &= 1.8094\ m \end{aligned}$$

BEMESS calculates the ψ value by using LoA (Level of Approximation) III. LoA I is used only at beginning of the calculation for iteration, when:

$$\frac{m_{sdx}}{m_{Rd}} = 1.$$

Rotation ψ; 4.3.6.4.1 and 4.3.6.4.2; Eq. (59)

For Level of Approximation III:

$$\begin{aligned} \psi_x &= 1.2 \cdot \frac{r_{sx}}{d} \cdot \frac{f_{sd}}{E_s} \cdot \left(\frac{m_{sdx}}{m_{Rd}}\right)^{3/2} \\ &= 1.2 \cdot \frac{1.166}{0.204} \cdot \frac{434.78}{205000} \cdot \left(\frac{105.53}{112.306}\right)^{3/2} \\ &= 1.325\ \% \end{aligned}$$

$$\begin{aligned} \psi_y &= 1.2 \cdot \frac{r_{sy}}{d} \cdot \frac{f_{sd}}{E_s} \cdot \left(\frac{m_{sdy}}{m_{Rd}}\right)^{3/2} \\ &= 1.2 \cdot \frac{1.248}{0.204} \cdot \frac{434.78}{205000} \cdot \left(\frac{105.85}{112.306}\right)^{3/2} \\ &= 1.424\ \% \end{aligned}$$

The governing value is $\psi = max(\psi_x, \psi_y) = 1.424\ \%$

k_r; 4.3.6.3.2; Eq. (58)

The coefficient k_r:

$$\begin{aligned} k_r &= \frac{1}{0.45 + 0.18 \cdot \psi \cdot d \cdot k_g} \\ &= \frac{1}{0.45 + 0.18 \cdot \psi \cdot d \cdot k_g} \\ &= \frac{1}{0.45 + 0.18 \cdot 0.01424 \cdot 204 \cdot 1} \\ &= 1.02786 \end{aligned}$$

In BEMESS there isn't any option to set the D_{max} value and the D_{max} is strictly defined:

- For normal concrete $D_{max} = 32\ mm \rightarrow k_g = 1.0$
- For high-strength and lightweight concrete, the $D_{max} = 0 \rightarrow k_g = 3$.

STEP 6: Punching strength with and without shear reinforcement

According to SIA 262:2013 the punching strength without shear reinforcement is calculated: 4.3.6.3.1; Eq. (57)

$$\begin{aligned} V_{Rd,c} &= k_r \cdot \tau_{cd} \cdot d_v \cdot u \\ &= 1.02786 \cdot 1.095 \cdot 0.204 \cdot 1.5129 \\ &= 0.34736\ MN\ =\ 347.36\ KN \end{aligned}$$

$V_d > V_{Rd,c} \rightarrow$ shear/punching reinforcement is necessary

Calculating the maximum punching strength $V_{Rd,max}$ 4.3.6.5.7; Eq. (69)

$$\begin{aligned} V_{Rd,max} &= 2 \cdot k_r \cdot \tau_{cd} \cdot d_v \cdot u_{red} \leq 3.5 \cdot \tau_{cd} \cdot d_v \cdot u_{red} \\ &= 2 \cdot 1.0278 \cdot 1.095 \cdot 0.204 \cdot 1.5129 \leq 3.5 \cdot 1.095 \cdot 0.204 \cdot 1.5129 \\ &= 0.69469\ MN \leq 1.182\ MN \\ &= 694.69\ kN \end{aligned}$$

Punching strength with shear reinforcement: 4.3.6.5; Eq. (65) and Eq. (66)

$$V_{d,s} = max(V_d - V_{Rd,c};\ 0.5 \cdot V_d)$$

$$V_d - V_{Rd,c} = 686.1 - 347.36 = 338.74\ kN$$

$$V_d \cdot 0.5 = 686.1 \cdot 0.5 = 343.05\ kN$$

$$V_{d,s} = max(338.74;\ 343.05) = 343.05\ kN$$

Calculating the reinforcement:

$$
\begin{aligned}
sin\beta \cdot A_{sw} &= \frac{V_{d,s}}{k_e \cdot \sigma_{sd}} \\
&= \frac{343.05}{0.9 \cdot 43.478} \\
&= \underline{\mathbf{8.766}\ cm^2}
\end{aligned}
$$

4.3.6.5.4; Eq. (68)

where σ_{sd} is calculated according:

$$
\begin{aligned}
\sigma_{sd} &= \frac{E_s \cdot \psi}{6} \cdot \left(1 + \frac{f_{bd}}{f_{sd}} \cdot \frac{d}{\phi_{sw}}\right) \leq f_{sd} \\
&= \frac{205000 \cdot 0.01424}{6} \cdot \left(1 + \frac{2.703}{434.78} \cdot \frac{204}{16}\right) \leq 434.78 \\
&= 487.00 \cdot (1 + 0.006216 \cdot 12.75) \leq 434.78 \\
&= 525.60 \leq 434.78 \\
&= 434.78\ N/mm^2
\end{aligned}
$$

Please note that in BEMESS, $sin\beta = 1.0$.

STEP 7: Failure outside the shear reinforcement

To avoid failure outside the shear reinforcement area, BEMESS iterates the perimeter until the shear strength $V_{Rdc} \geq V_d$.

In this example, the calculating value of the effective depth d_v is equal to the effective depth d minus the distance from concrete cover c on the bottom surface of the slab: $d_{v,out} = d - c = 204 - 40 = 164\ mm$

$$V_{Rd,c,out} = 2 \cdot k_r \cdot \tau_{cd} \cdot d_{v,out} \cdot u_{out} = V_d$$

$$u_{out} = \frac{0.6861}{1.02786 \cdot 1.095 \cdot 0.164} = 3.7170\ m$$

$$r_{out} = \frac{u_{out} - 4 \cdot b_c}{2 \cdot \pi} = \frac{3.717 - 4 \cdot 0.26}{2 \cdot \pi} = 0.426\ m$$

5 Conclusion

This example shows the calculation of punching of flat slabs and it has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *SIA 262:2013 Concrete Structures*. Swiss Society of Engineers and Architects. 2013.
[2] S. Lips, A. Muttoni, and M.F. Ruiz. "Punching of flat slabs: Design Example". In: *fib Model Code 2010* (2011).
[3] fib Model Code 2010. *fib Model Code for Concrete Structures 2010*. International Federation for Structural Concrete (fib). 2010.
[4] A. Muttoni et al. *Background to fib Model Code 2010 shear provisions - part II: punching shear*. Tech. rep. fib, 2013.