Benchmark Example No. 1

Punching of flat slabs acc. SIA 262

SOFiSTiK | 2018
1 Problem Description

The problem consists of a flat slab. The structure under consideration is a five-storey residential building with the geometry and main dimensions given in Fig. 1. The design of slab against punching at the columns is discussed in the following.

For the concrete, strength class C30/37 ($f_{ck} = 30$ MPa, $\gamma_c = 1.5$) is assumed, for the reinforcing steel, grade B500B ($f_{yk} = 500$ MPa, $E_S = 205$ GPa, $\gamma_S = 1.15$, ductility class B). The factored design load accounting for self-weight, dead load and imposed load is $q_d = 15.6 \text{kN/m}^2$.

The width of the slab is $h = 26 \text{ cm}$ as shown in Fig. 2.

(a) SOFiSTiK Model

Figure 1: View of building

Figure 2: Section through flat slab and supporting columns
This example is concerned with the punching of flat slabs. The content of this problem is covered by the following parts of SIA 262:2013 [1]:

- Construction materials (Section 2.2)
- Dimensoning values (Section 4.2)
- Shear force (Section 4.3.3)
- Punching (Section 4.3.6)
3 Model and Results

The goal of the preliminary design is to check if the dimensions of the structure are reasonable with respect to the punching shear strength and if punching shear reinforcement is needed.

In the reference example the reaction forces are estimated by using contributive areas. The results are given in Table 2.

<table>
<thead>
<tr>
<th>Result</th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner column C5 (Node 1070)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>1.681 m</td>
<td>1.681 m</td>
</tr>
<tr>
<td>$k_e$</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$u_{red}$</td>
<td>1.5129 m</td>
<td>1.5129 m</td>
</tr>
<tr>
<td>$\psi_x$</td>
<td>1.33 %</td>
<td>1.325 %</td>
</tr>
<tr>
<td>$\psi_y$</td>
<td>1.43 %</td>
<td>1.424 %</td>
</tr>
<tr>
<td>$k_r$</td>
<td>1.03</td>
<td>1.02786</td>
</tr>
<tr>
<td>$V_{Rd,c}$</td>
<td>346.6 kN</td>
<td>347.36 kN</td>
</tr>
<tr>
<td>$V_{Rd,c,max}$</td>
<td>693.2 kN</td>
<td>694.69 kN</td>
</tr>
<tr>
<td>$A_{sw}$</td>
<td>10.93 cm$^2$</td>
<td>8.766 cm$^2$</td>
</tr>
</tbody>
</table>
4 Design Process\(^1\)

The calculation steps of the reference solution are presented below.

**STEP 1:** Material

Concrete 30/37

\[ f_{ck} = 30 \, N/mm^2 \]

Conversion factor \( \eta_c \): SIA 262:2013; 4.2.1.2; Eq. (26)

\[ \eta_c = \left( \frac{30}{f_{ck}} \right)^{1/3} = \left( \frac{30}{30} \right)^{1/3} = 1.00 \leq 1.00 \]

The dimensioning value of the concrete compressive strength; 2.3.2.3; Eq. (2)

\[ f_{cd} = \eta_c \cdot f_{ck} = \frac{1.1 \cdot 30}{1.5} = 20 \, N/mm^2 \]

The dimensioning value of shear stress limit; 2.3.2.4; Eq. (3)

\[ \tau_{cd} = \frac{0.3 \cdot \eta_c \cdot \sqrt{f_{ck}}}{\gamma_c} = \frac{0.3 \cdot 1.0 \cdot \sqrt{30}}{1.5} = 1.095 \, N/mm^2 \]

\[ D_{max} = 32 \, mm \]

**STEP 2:** Reinforcement

Steel B500B (flexural and transverse reinforcement)

\[ f_{yd} = 435 \, MPa \]

\[ E_s = 205000 \, MPa \]

Ductility class: B

For \( \phi 10 @200 \, mm/\phi 16 @200 \, mm \)

**STEP 3:** Cross-section

\[ d = h - d_1 \]

\[ = 26 - \left( 4.8 + \frac{1.6}{2} \right) \]

\[ = 20.4 \, cm \]

**STEP 4:** Calculating the control perimeter

Inner:

\[ u = 2 \cdot a + 2 \cdot b + d_v \cdot \pi \]

\[ = 2 \cdot 26 + 2 \cdot 26 + 20.4 \cdot 3.14 \]

\[ = 168.1 \, cm \]

In BEMESS the shear force \( V_d \) is equal to column reaction force minus the applied load within the control perimeter \( g_d \cdot A_c \). The value \( q_d \) is not taken into account.

\[ V_d = 686.1 \, kN \]

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\(^1\)The sections mentioned in the margins refer to SIA 262:2013 [1] unless otherwise specified.
Bemess takes into account the min. value of $k_e$.

$$k_e = \min \left( 0.9; 1 + \frac{1}{\frac{e_u}{b}} \right)$$

(1)

$$e_u = \left| \frac{M_d}{V_d} - \Delta e \right| = \left| \frac{5.46 \, kNm}{686.1 \, kN} \right| = 7.95 \, mm$$

In case of inner columns, the centroid of the column corresponds to the centroid of the control perimeter. Therefore, $\Delta e = 0$

$$A_c = b_c^2 + 4 \cdot b_c \cdot \frac{d_v}{2} + \frac{d_v^2}{4} \cdot \pi$$

$$= 0.26^2 + 4 \cdot 0.26 \cdot \frac{0.204}{2} + \frac{0.204^2}{4} \cdot \pi$$

$$= 0.2063 \, m^2$$

$$b_u = \sqrt{\frac{4}{\pi} \cdot A_c} = \sqrt{\frac{4}{\pi} \cdot 0.2063} = 0.5129 \, m = 512.9 \, mm$$

According to Eq. 1:

$$k_e = \min \left( 0.9; 1 + \frac{1}{\frac{e_u}{b}} \right)$$

$$= \min \left( 0.9; 1 + \frac{1}{\frac{0.204}{b}} \right)$$

$$= \min \left( 0.9; 1 + \frac{1}{\frac{7.95}{512.9}} \right)$$

$$= \min (0.9, 0.98)$$

$$= 0.9$$

Where $e_u$ is the eccentricity of the resultant of shear forces with respect to the centroid of the basic control perimeter and $b_u$ is the diameter of a circle with the same surface as the region inside the basic control perimeter.

Reduced control perimeter is calculated:

$$u_{red} = k_e \cdot u = 0.9 \cdot 168.1 = 151.29 \, cm$$

**STEP 5: Rotations**

The distances $r_{s,x}$ and $r_{s,y}$ are calculated from the results of the flexural analysis, one can obtain the distances between the center of the column
and the point, at which the bending moments are zero.

\[ r_{sx} = 1.166 \text{ m} \]

\[ r_{sy} = 1.248 \text{ m} \]

The average moment of the strip is calculated by the integration of the moments at the strip section. Since the flexural moments \( m_{d,x} \) and \( m_{d,y} \) are negative, the absolute value of the twisting moment \( m_{d,x} \) needs to be subtracted so that the absolute value of \( m_{sd,x} \) and \( m_{sd,y} \) will be maximized:

\[ m_{sd,x} = m_{d,x} - |m_{d,xy}| \]

\[ m_{sd,y} = m_{d,y} - |m_{d,xy}| \]

By using FEM analysis:

\[ m_{sd,x} = 105.53 \text{ kNm/m} \]

\[ m_{sd,y} = 105.81 \text{ kNm/m} \]

The representative width is calculated:

\[ b_s = 1.5 \cdot \sqrt{r_{sx} \cdot r_{sy}} \leq l_{\text{min}} \]

\[ b_s = 1.5 \cdot \sqrt{1.248 \cdot 1.166} \]

\[ b_s = 1.8094 \text{ m} \]

BEMESS calculates the \( \psi \) value by using LoA (Level of Approximation) III. LoA I is used only at beginning of the calculation for iteration, when:

\[ \frac{m_{sx}}{m_{Rd}} = 1. \]

For Level of Approximation III:

\[ \psi_x = 1.2 \cdot \frac{r_{sx}}{d} \cdot \frac{f_{sd}}{E_s} \cdot \left( \frac{m_{sd,x}}{m_{Rd}} \right)^{3/2} \]

\[ \psi_x = 1.2 \cdot \frac{1.166}{0.204} \cdot \frac{434.78}{205000} \cdot \left( \frac{105.53}{112.306} \right)^{3/2} \]

\[ \psi_x = 1.325 \% \]

\[ \psi_y = 1.2 \cdot \frac{r_{sy}}{d} \cdot \frac{f_{sd}}{E_s} \cdot \left( \frac{m_{sd,y}}{m_{Rd}} \right)^{3/2} \]

\[ \psi_y = 1.2 \cdot \frac{1.248}{0.204} \cdot \frac{434.78}{205000} \cdot \left( \frac{105.85}{112.306} \right)^{3/2} \]

\[ \psi_y = 1.424 \% \]

The governing value is \( \psi = \max(\psi_x, \psi_y) = 1.424 \% \)

The coefficient \( k_r \):
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\[
k_r = \frac{1}{0.45 + 0.18 \cdot \psi \cdot d \cdot k_g}
= \frac{1}{0.45 + 0.18 \cdot \psi \cdot d \cdot k_g}
= \frac{1}{0.45 + 0.18 \cdot 0.01424 \cdot 204 \cdot 1}
= 1.02786
\]

In BEMESS there isn’t any option to set the \(D_{\text{max}}\) value and the \(D_{\text{max}}\) is strictly defined:

- For normal concrete \(D_{\text{max}} = 32\) mm \(\rightarrow k_g = 1.0\)
- For high-strength and lightweight concrete, the \(D_{\text{max}} = 0 \rightarrow k_g = 3.0\)

**STEP 6: Punching strength with and without shear reinforcement**

According to SIA 262:2013 the punching strength without shear reinforcement is calculated:

\[
V_{Rd,c} = k_r \cdot \tau_{cd} \cdot d_v \cdot u
= 1.02786 \cdot 1.095 \cdot 0.204 \cdot 1.5129
= 0.34736 \text{ MN} = 347.36 \text{ KN}
\]

\(V_d > V_{Rd,c} \rightarrow \text{shear/punching reinforcement is necessary}\)

Calculating the maximum punching strength \(V_{Rd,max}\)

\[
V_{Rd,max} = 2 \cdot k_r \cdot \tau_{cd} \cdot d_v \cdot u_{red} \leq 3.5 \cdot \tau_{cd} \cdot d_v \cdot u_{red}
= 2 \cdot 1.0278 \cdot 1.095 \cdot 0.204 \cdot 1.5129 \leq 3.5 \cdot 1.095 \cdot 0.204 \cdot 1.5129
= 0.69469 \text{ MN} \leq 1.182 \text{ MN}
= 694.69 \text{ kN}
\]

Punching strength with shear reinforcement:

\[
V_{d,s} = \max(V_d - V_{Rd,c}; 0.5 \cdot V_d)
\]

\(V_d - V_{Rd,c} = 686.1 - 347.36 = 338.74 \text{ kN}\)

\(V_d \cdot 0.5 = 686.1 \cdot 0.5 = 343.05 \text{ kN}\)

\(V_{d,s} = \max(338.74; 343.05) = 343.05 \text{ kN}\)
Calculating the reinforcement:

\[
\sin \beta \cdot A_{sw} = \frac{V_{d,s}}{k_e \cdot \sigma_{sd}} = \frac{343.05}{0.9 \cdot 43.478} = 8.766 \text{ cm}^2
\]

where \( \sigma_{sd} \) is calculated according:

\[
\sigma_{sd} = \frac{E_s \cdot \psi}{6} \left( 1 + \frac{f_{bd}}{f_{sd}} \cdot \frac{d}{\phi_{sw}} \right) \leq f_{sd}
\]

\[
= \frac{205000 \cdot 0.01424}{6} \left( 1 + \frac{2.703}{434.78} \cdot \frac{204}{16} \right) \leq 434.78
\]

\[
= 487.00 \cdot (1 + 0.006216 \cdot 12.75) \leq 434.78
\]

\[
= 525.60 \leq 434.78
\]

\[
= 434.78 \text{ N/mm}^2
\]

Please note that in BEMESS, \( \sin \beta = 1.0 \).

**STEP 7:** Failure outside the shear reinforcement

To avoid failure outside the shear reinforcement area, BEMESS iterates the perimeter until the shear strength \( V_{Rdc} \geq V_d \).

In this example, the calculating value of the effective depth \( d_v \) is equal to the effective depth \( d \) minus the distance from concrete cover \( c \) on the bottom surface of the slab: \( d_{v, out} = d - c = 204 - 40 = 164 \text{ mm} \)

\[
V_{Rd,c, out} = 2 \cdot k_r \cdot \tau_{cd} \cdot d_{v, out} \cdot u_{out} = V_d
\]

\[
u_{out} = \frac{0.6861}{1.02786 \cdot 1.095 \cdot 0.164} = 3.7170 \text{ m}
\]

\[
r_{out} = \frac{u_{out} - 4 \cdot b_c}{2 \cdot \pi} = \frac{3.7170 - 4 \cdot 0.26}{2 \cdot \pi} = 0.426 \text{ m}
\]
5 Conclusion

This example shows the calculation of punching of flat slabs and it has been shown that the results are reproduced with excellent accuracy.

6 Literature