Benchmark Example No. 12

Cantilever in Torsion

SOFiSTiK | 2020


1 Problem Description

The problem consists of a cantilever beam as shown in Fig. 1. The tip of the cantilever is offsetted in y-direction by \( \Delta_y = l/200 = 2.5 \text{ cm} \), creating a geometrical imperfection. The beam is loaded with a transverse force \( P_z \) and an axial force \( P_x \). The imperfection acts as a lever arm for the loading, causing a torsional moment. The torsional moment at the support with respect to the local and global coordinate system is determined.

\[
M_{\text{global}} = P_z \left( u_y + \Delta_y \right) - P_y u_z , \tag{1}
\]

whereas by the local x-axis the torsional moment \( M_{\text{local}} \) is:

\[
M_{\text{local}} = P_z u_y + P_x \left( \frac{\Delta_y}{l} \right) u_z , \tag{2}
\]

where \( l \) is the length of the beam, \( \Delta_y \) the initial geometrical imperfection and \( P_x \) is negative for compression.

2 Reference Solution

In order to account for the effect of the geometrical imperfection on the structure, second-order theory should be used, where the equilibrium is established at the deformed system. According to the equilibrium of moments at the deformed system, with respect to the global x-axis, the torsional moment at the support \( M_{\text{global}} \) is:

\[
M_{\text{global}} = P_z \left( u_y + \Delta_y \right) - P_y u_z , \tag{1}
\]

3 Model and Results

The properties of the model [1] [2] are defined in Table 1. A standard steel material is used as well as a standard hot formed hollow section with properties according to DIN 59410, DIN EN 10210-2. A safety
factor $\gamma_M = 1.1$ is used, which according to DIN 18800-2 it is applied both to the yield strength and the stiffness. Furthermore, the self weight, the shear deformations and the warping modulus $C_M$ are neglected. At the support the warping is not constrained.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$ 355</td>
<td>$l = 5 \text{ m}$</td>
<td>$P_z = 10 \text{ kN}$</td>
</tr>
<tr>
<td>$\gamma_M = 1.1$</td>
<td>$RRo/SH \ 200 \times 100 \times 10$ [3]</td>
<td>$P_x = 100 \text{ kN}$</td>
</tr>
<tr>
<td>$C_M = 0$</td>
<td>$\Delta_y = 2.5 \text{ cm}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>$u_y$</th>
<th>$u_z$</th>
<th>$M_{x,\text{global}}$</th>
<th>$M_{x,\text{local}}$</th>
<th>$P_{\text{buck}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[cm]</td>
<td>[cm]</td>
<td>[kNcm]</td>
<td>[kNcm]</td>
<td>[kN]</td>
</tr>
<tr>
<td>SOF.</td>
<td>3.209</td>
<td>10.204</td>
<td>57.08</td>
<td>26.98</td>
<td>163.7</td>
</tr>
<tr>
<td>Ref.[4]</td>
<td>3.20</td>
<td>10.2</td>
<td>57.0</td>
<td>26.9</td>
<td>164</td>
</tr>
</tbody>
</table>

The corresponding results are presented in Table 2. Figure 2 shows the deformed shape of the structure and the nodal displacements for the $z$ and $y$ direction. From the presented results, we can observe that the values of the moments are correctly computed. Here has to be noted that the reference results are according to [1], where they are computed with another finite element software, and not with respect to an analytical solution.

Figure 2: Deformations [mm]
4 Conclusion

This example presents a case where torsion is induced to the system because of an initial geometrical imperfection. It has been shown that the behaviour of the beam is captured accurately.

5 Literature


