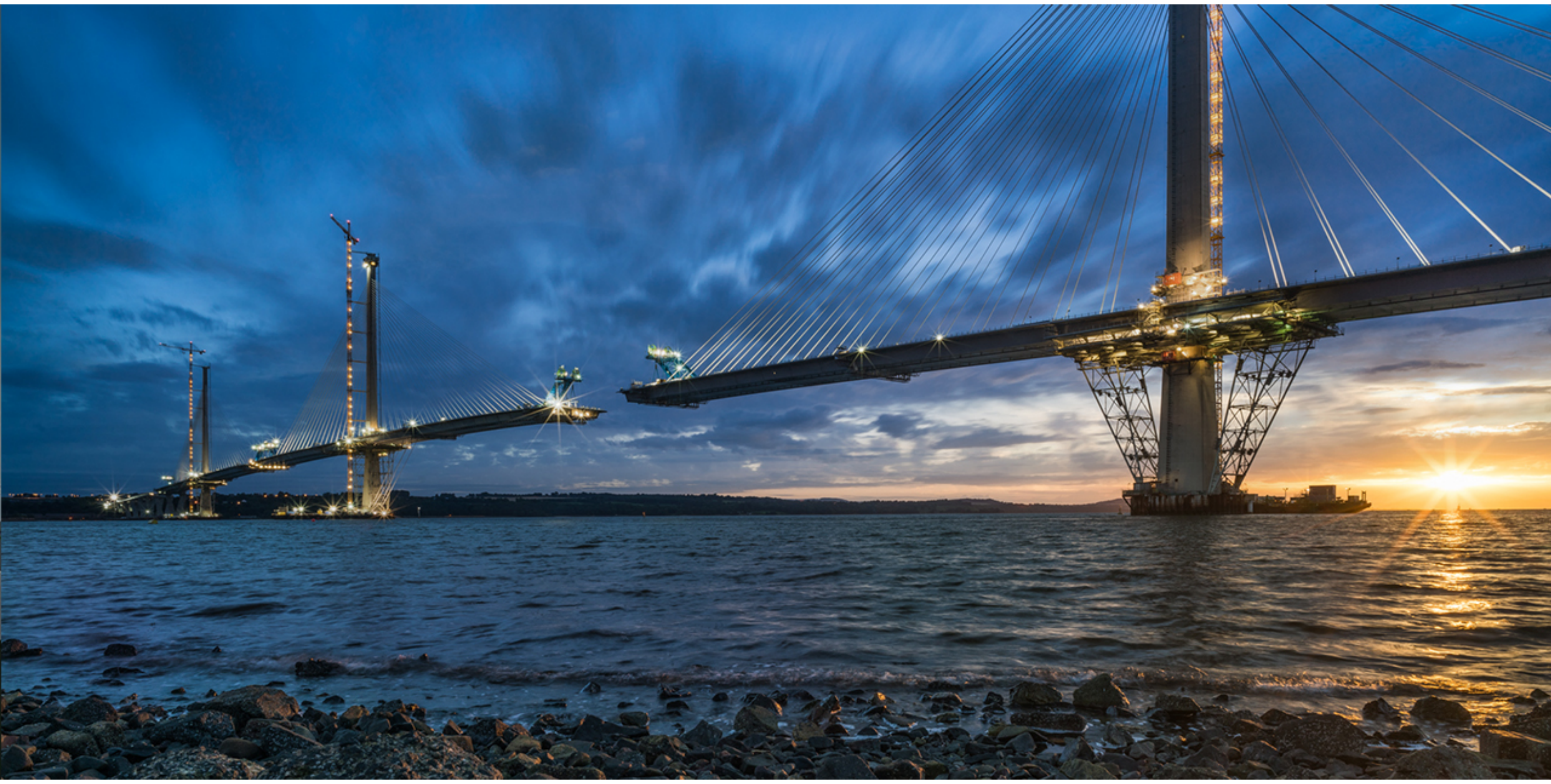

Benchmark Example No. 13

Buckling of a Bar with Hinged Ends I

SOFiSTiK | 2020

VERiFiCATiON
BE13 Buckling of a Bar with Hinged Ends I

VERiFiCATiON Manual, Service Pack 2020-14 Build 134

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Front Cover

Project: Queensferry Crossing | Photo: Bastian Kratzke

Overview	
Element Type(s):	B3D
Analysis Type(s):	STAT, GNL
Procedure(s):	STAB
Topic(s):	
Module(s):	ASE
Input file(s):	buckling_bar.dat

1 Problem Description

The problem consists of an axially loaded long slender bar of length l with hinged ends, as shown in Fig. 1. Determine the critical buckling load. [1]

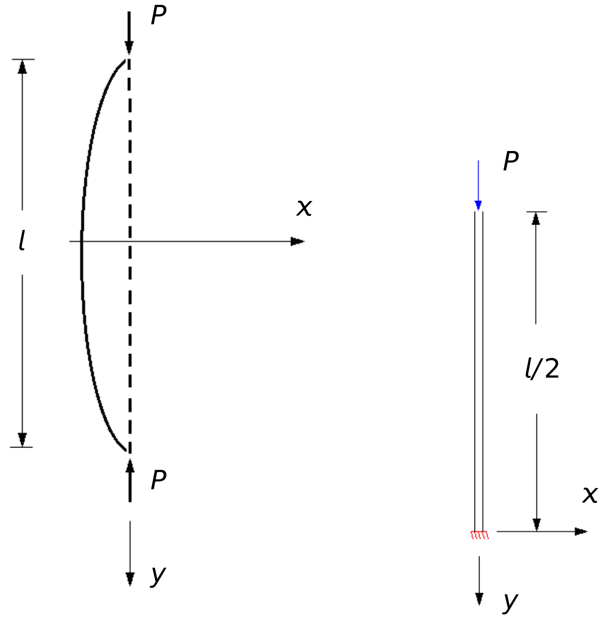

Figure 1: Problem Description

2 Reference Solution

The problem of lateral buckling of bars is examined here. The case of a bar with hinged ends is very often encountered in practical applications and is called the $fundamental$ case of buckling of a prismatic bar. For the case of an axially compressed bar there is a certain critical value of the compressive force at which large lateral deflection may be produced by the slightest lateral load. For a prismatical bar with hinged ends (Fig. 1) this critical compressive force is [2]:

$$P_{cr} = \frac{\pi^2 EI}{(\beta l)^2} = \frac{\pi^2 EI}{l^2}, \tag{1}$$

where l is the full length of the bar, EI its flexural rigidity and β the effective length coefficient, whose value depends on the conditions of end support of the bar. For the fundamental case, $\beta = 1$. If the load P is less than its critical value the bar remains straight and undergoes only axial compression.

This straight form of elastic equilibrium is stable, i.e., if a lateral force is applied and a small deflection is produced this deflection disappears when the lateral load is removed and the bar becomes straight again. By increasing *P* up to the critical load causes the column to be in a state of unstable equilibrium, which means, that the introduction of the slightest lateral force will cause the column to undergo large lateral deflection and eventually fail by buckling.

3 Model and Results

Only the upper half of the bar is modelled because of symmetry (Fig. 1). The boundary conditions thus become free-fixed for the half symmetry model. A total of 20 elements are used to capture the buckling mode. The properties of the model are defined in Table 1.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 300\,MPa$	$l = 20\,m$	$P_y = 1\,kN$
	$h = 0.5\,m$	$P_x << 1\,kN$
	$A = 0.25\,m^2$	
	$I = 5.20833 \times 10^{-3}\,m^4$	
	$\beta = 2$, free-fixed ends	

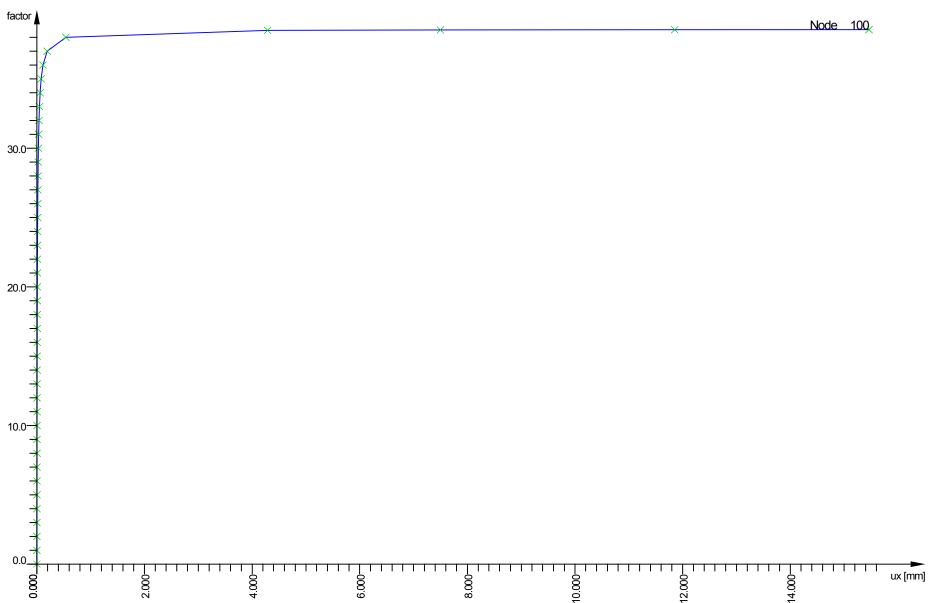

Figure 2: Load-Deflection curve

A small horizontal load at the top is necessary in order to induce an initial horizontal displacement. It should be sufficiently large to cause a nonlinear iteration, but it should not affect the result unintentionally. A buckling eigenvalue determination is performed where the critical load factor is calculated. The results are presented in Table 2. Moreover, an ultimate limit load iteration is done and the produced Load-Deflection curve is shown in Fig. 2, as well as a part of the iteration summary.

Table 2: Results

	SOF.	Ref.
P_{cr} $[kN]$	38.553	38.553

4 Conclusion

This example presents the buckling of slender bars. It has been shown that the buckling properties of the bar are accurately captured.

5 Literature

[1] *Verification Manual for the Mechanical APLD Application, Release 12.0*. Ansys, Inc. 2009.
[2] S. Timoshenko. *Strength of Materials, Part II, Advanced Theory and Problems*. 2nd. D. Van Nostrand Co., Inc., 1940.