Benchmark Example No. 51

Pushover Analysis: Performance Point Calculation by EC8 Procedure
1 Problem Description

The following example is intended to verify the Eurocode 8 (EC8) procedure for the calculation of the performance point (illustrated schematically in Fig. 1), as implemented in SOFiSTiK. The elastic demand and capacity diagrams are assumed to be known.

2 Reference Solution

The reference solution is provided in [1].

Assuming that the elastic demand diagram (5% elastic response spectrum in ADRS format\(^1\)) and the capacity diagram are known, it is possible to determine the performance point \(PP \left( S_{dp}, S_{ap} \right) \) (Fig. 1). The procedure comprises of a series of trial calculations (trial performance points \(PP_t \left( S_{dp}, t, S_{ap}, t \right) \)), in which the equivalent inelastic single degree of freedom system (SDOF), represented by the capacity diagram, is idealized with the equivalent inelastic SDOF system with a bi-linear force-deformation relationship. The response in form of the performance point \(PP\) is then calculated from the inelastic response spectrum (demand diagram). The computation stops when the performance point \(PP\) is within a tolerance of a trial performance point \(PP_t\). Detailed description of this procedure can be found in [2], [3], [1] and [4].

In the reference example [1] the bi-linear idealization of the capacity is assumed to be independent of the performance point and it is performed at the beginning of the analysis. This eliminates the need for the iterations and the solution of the problem can be obtained in a single calculation step.

Hence in this example it is assumed that the bi-linear idealization of the capacity diagram is already known, which means that the point \(PY \left( S_{dy}, S_{ap} \right) \) is given. The procedure to calculated the performance point is illustrated in Fig. 2 and can be summarized as follows [4]:

\(^1\)ADRS = Spectral Acceleration \(S_a\) - Spectral Displacement \(S_d\) format
1. Determine the period of the idealized system $T^* = T_y$ from the known $PY (S_{dy}, S_{ay})$:

$$T^* = T_y = 2\pi \cdot \sqrt{\frac{S_{dy}}{S_{ay}}};$$  

(1)

2. Calculate the elastic spectral response $PE (S_{de}, S_{ae})$ of the idealized equivalent SDOF system with the period $T^* = T_y$ from the given 5%-damped elastic response spectrum (Fig. 2);

3. Calculate the yield strength reduction factor $R_y$:

$$R_y = \frac{S_{ae}}{S_{ay}};$$  

(2)

4. Calculate ductility $\mu$:

$$\mu = \begin{cases} 
(R_y - 1) \cdot \frac{T_C}{T^*} + 1 & \text{for } T^* < T_C \\
R_y & \text{for } T^* \geq T_C 
\end{cases};$$

(3)

5. Determine the performance point $PP (S_{dp}, S_{ap})$ from the inelastic design spectrum:

$$S_{dp} = \mu \cdot S_{dy} = \mu \cdot \frac{S_{de}}{R_y},$$  

(4a)

$$S_{ap} = \frac{S_{ae}(T^*)}{R_y}.$$

(4b)

3 Model and Results

In order to verify the analysis procedure for the determination of the performance point, a test case has been set up in such a way that it comprises of a SDOF with a unit mass and a non-linear spring element. It is obvious that for such an element the quantities governing the transformation from the original system
to the equivalent inelastic SDOF system must be equal to one, i.e.

\[ \phi_{cnod} = 1 \ ; \ \Gamma = 1 \ ; \ m = 1, \]  

(5)

where \( \phi_{cnod} \) is the eigenvector value at control node, \( \Gamma \) is the modal participation factor and \( m \) is the generalized modal mass. Writing now the equations which govern the conversion of the pushover curve to capacity diagram, we obtain [4]

\[ S_d = \frac{u_{cnod}}{\phi_{cnod} \cdot \Gamma} = u_{cnod}, \]  

(6a)

\[ S_a = \frac{V_b}{\Gamma^2 \cdot m} = V_b, \]  

(6b)

where \( V_b \) is the base shear and \( u_{cnod} \) is the control node displacement.

Since the original system is a SDOF system, \( V_b \) and \( u_{cnod} \) are nothing else but the force in spring \( P \) and the displacement of the unit mass \( u \), respectively. It follows further that the force-displacement work law assigned to the spring element corresponds to the capacity diagram in ADRS format, with the force \( P \) and displacement \( u \) equal to \( S_a \) and \( S_d \), respectively.

The bi-linear idealization of the capacity diagram used in the reference example is defined by two points, whose coordinates are listed in the Table 1 [2]. According to the analysis above, these points can be used to define the force-displacement work law \( P - u \) of the non-linear spring element (Fig. 3).

Table 1: Model Properties [1]

<table>
<thead>
<tr>
<th>Capacity Diagram</th>
<th>Elastic Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point ( (S_d[mm], S_a[m/s^2]) )</td>
<td>5%-Damped Elastic Response Spectrum</td>
</tr>
<tr>
<td>( A ) ( (61, 3.83) )</td>
<td>( a_g = {0.60g, 0.30g, 0.16g} )</td>
</tr>
<tr>
<td>( B ) ( (\infty, 3.83) )</td>
<td>( S_A = 1.0, S_B = 2.5, k_1 = 1.0 )</td>
</tr>
<tr>
<td></td>
<td>( T_B = 0.15s, T_C = 0.60s, T_D = 3.00s )</td>
</tr>
</tbody>
</table>

Figure 3: Force-displacement work law of the non-linear spring

The elastic demand is a 5%-damped elastic response spectrum, whose properties are summarized in Table 1. Three levels of peak ground acceleration \( a_g \) have been taken into account. The shape of

2Not that the point \( A \) is nothing else but the point \( P \) \( (S_{dy}, S_{ay}) \).
the spectrum and the meaning of the parameters specified in Table 1 are shown in Figure 4.

\[
\begin{align*}
0 \leq T \leq T_B : & \quad S = S_A + \frac{T}{T_B} \cdot (S_B - S_A) \\
T_B \leq T \leq T_C : & \quad S = S_B \\
T_C \leq T \leq T_D : & \quad S = S_B \cdot \left(\frac{T_C}{T}ight)^{k_1}
\end{align*}
\]

Figure 4: 5%-Damped Elastic Response Spectrum (El. Demand Diagram)

The outcome of the analysis is shown in Figures 5 to 7.

Figure 5: Capacity-Demand-Diagram \((\alpha_g = 0.60g)\)

Figure 6: Capacity-Demand-Diagram \((\alpha_g = 0.30g)\)
The results of the SOFiSTiK calculation and the comparison with the reference solution are summarized in Table 2.

Table 2: Results

<table>
<thead>
<tr>
<th>$a_g$ [g]</th>
<th>$\mu$</th>
<th>$R_{yp}$</th>
<th>$T_y$ [s]</th>
<th>$S_{dy}$ [mm]</th>
<th>$S_{dp}$ [mm]</th>
<th>$S_{ap}$ [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOF.</td>
<td>2.9</td>
<td>2.9</td>
<td>0.79</td>
<td>61</td>
<td>177</td>
<td>3.83</td>
</tr>
<tr>
<td>0.60 Ref. [1]</td>
<td>2.9</td>
<td>2.9</td>
<td>0.79</td>
<td>61</td>
<td>177</td>
<td>3.83</td>
</tr>
<tr>
<td>$</td>
<td>e</td>
<td>[%]$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>SOF.</td>
<td>1.5</td>
<td>1.5</td>
<td>0.79</td>
<td>61</td>
<td>89</td>
<td>3.83</td>
</tr>
<tr>
<td>0.30 Ref. [1]</td>
<td>1.5</td>
<td>1.5</td>
<td>0.79</td>
<td>61</td>
<td>89</td>
<td>3.83</td>
</tr>
<tr>
<td>$</td>
<td>e</td>
<td>[%]$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>SOF.</td>
<td>1.0</td>
<td>1.0</td>
<td>0.79</td>
<td>44</td>
<td>44</td>
<td>2.78</td>
</tr>
<tr>
<td>0.15 Ref. [1]</td>
<td>1.0</td>
<td>1.0</td>
<td>0.79</td>
<td>44</td>
<td>44</td>
<td>2.76</td>
</tr>
<tr>
<td>$</td>
<td>e</td>
<td>[%]$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

$\mu$  displacement ductility factor  
$R_{yp}$  reduction factor due to ductility at performance point  
$T_y$  period associated with yielding point  
$S_{dy}, S_{dp}$  spectral displacements at yielding and performance point  
$S_{ap}$  pseudo spectral acceleration at performance point

The results are in excellent agreement with the reference solution.
4 Conclusion

Excellent agreement between the reference and the results computed by SOFiSTiK verifies that the procedure for the calculation of the performance point according to Eurocode 8 is adequately implemented.

5 Literature