Benchmark Example No. 15

Buckling Resistance of Steel Members

SOFiSTiK | 2020
1 Problem Description

The problem consists of a simply supported beam with a steel I-section subject to uniform end moments, as shown in Fig. 1. The cross-section is checked against buckling.

![Diagram of a simply supported beam with a steel I-section subject to uniform end moments](image)

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the buckling resistance of steel members. Lateral torsional buckling occurs in unrestrained, or inadequately restrained beams bent about the major axis and this causes the compression flange to buckle and deflect sideways, thus inducing twisting of the section. The content of this problem is covered by the following parts of EN 1993-1-1:2005 [1]:

- Structural steel (Section 3.2)
- Classification of cross-sections (Section 5.5)
- Buckling resistance of members (Section 6.3)

3 Model and Results

The I-section, a UB 457x152x74, with properties as defined in Table 1, is to be checked for buckling, with respect to EN 1993-1-1:2005 [1]. The calculation steps [2] are presented below and the results are given in Table 2.
### Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>S 275 $E = 210000 , N/mm^2$</td>
<td>L $= 8 , m$ UB 457x152x74</td>
<td>$M_y = 150 , kNm$</td>
</tr>
<tr>
<td>$h = 462.0 , mm$</td>
<td>$b = 154.4 , mm$</td>
<td></td>
</tr>
<tr>
<td>$t_f = 17.0 , mm$</td>
<td>$t_w = 9.6 , mm$</td>
<td></td>
</tr>
<tr>
<td>$r = 10.2 , mm$</td>
<td>$A = 94.48 , cm^2$</td>
<td></td>
</tr>
<tr>
<td>$I_z = 1046.5 , cm^4$</td>
<td>$I_T = 66.23 , cm^4$</td>
<td></td>
</tr>
<tr>
<td>$I_w = 516297.12 , cm^6$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{cr} , [kNm]$</td>
<td>154.17</td>
<td>154.26</td>
</tr>
<tr>
<td>$\bar{\lambda}_{LT}$</td>
<td>1.703</td>
<td>1.703</td>
</tr>
<tr>
<td>$\Phi_{LT}$</td>
<td>1.907</td>
<td>1.907</td>
</tr>
<tr>
<td>$\chi_{LT}$</td>
<td>0.321</td>
<td>0.321</td>
</tr>
<tr>
<td>$M_{Ed}/M_{b,Rd} , (BDK)$</td>
<td>1.045</td>
<td>1.045</td>
</tr>
</tbody>
</table>
4 Design Process

Material:
Structural Steel S 275

\( f_y = 275 \text{ N/mm}^2 \) for maximum thickness \( \leq 40 \text{ mm} \)
\( \epsilon = \sqrt{235/f_y} = 0.924 \)

Design Load:
\( M_{Ed} = 150kNm \)

The cross-section is classified as Class 1, as demonstrated in Benchmark DCE-EN14.

\[
M_{c,Rd} = M_{pl,Rd,y} = \frac{W_{pl,y} \cdot f_y}{\gamma_{M0}} = 447.31 \text{ kNm}
\]

\[
M_{cr} = \frac{\pi \sqrt{Ez}}{G} \left( 1 + \frac{\pi^2 EI_w}{GL^2} \right)
\]

\[
M_{cr} = \frac{3.14 \cdot \sqrt{2197.74 \cdot 53.496}}{8} \sqrt{\left( 1 + \frac{3.14^2 \cdot 108.42}{53.496 \cdot 8^2} \right)}
\]

\[
M_{cr} = 154.26 \text{ kNm}
\]

\[
\lambda_{LT} = \sqrt{\frac{W_{pl,y} f_y}{M_{cr}}} = \sqrt{\frac{447.31}{154.26}} = 1.703
\]

\[
\Phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT} - \lambda_{LT,0} + \beta \lambda_{LT}^2 \right) \right]
\]

\[
h / b = 462 / 154.4 = 2.99 > 2
\]

for rolled I-sections and \( h / b > 2 \rightarrow \) buckling curve c

for buckling curve c \( \rightarrow \alpha_{LT} = 0.49\)

\[
\Phi_{LT} = 0.5 \left[ 1 + 0.49 \left( 1.703 - 0.4 + 0.75 \cdot 1.703^2 \right) \right] = 1.907
\]

\[
\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \lambda_{LT}^2}} = 0.321
\]

but \( \chi_{LT} = 0.321 \leq 1.0 \) and \( \chi_{LT} = 0.321 \leq \frac{1}{\lambda_{LT}^2} = 0.345 \)

\[
M_{b,Rd} = \chi_{LT} W_{pl,y} \frac{f_y}{\gamma_{M1}} = 143.587 \text{ kNm}
\]

\[
\frac{M_{Ed}}{M_{b,Rd}} = 1.045 \rightarrow \text{Beam fails in LTB}
\]
5 Conclusion

This example shows the check for lateral torsional buckling of steel members. It has been shown that the results are reproduced with excellent accuracy.

6 Literature
