Benchmark Example No. 17

Stress Calculation at a Rectangular Prestressed Concrete CS

SOFiSTiK | 2020
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The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
1 Problem Description

The problem consists of a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The stresses developed at the section due to prestress and bending are verified.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the design of prestressed concrete cross-sections, subject to bending and prestress force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Stress-strain curves for concrete and prestressing steel (Section 3.1.7, 3.3.6)
- Verification by the partial factor method - Design values (Section 2.4.2)
- Prestressing force (Section 5.10.2, 5.10.3)

![Figure 2: Stress Distribution in Prestress Concrete Cross-section](image)

In rectangular cross-sections, which are prestressed and loaded, stress conditions are developed, as shown in Fig. 2, where the different contributions of the loadings can be seen. The design stress-strain diagrams for prestressing steel is presented in Fig. 3, as defined in DIN EN 1992-1-1:2004 [1] (Section 3.3.6).
3 Model and Results

The simply supported beam of Fig. 4, consists of a rectangular cross-section with properties as defined in Table 1 and is prestressed and loaded with its own weight. A verification of the stresses is performed in the middle of the span with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2]. The geometry of the tendon can be visualised in Fig. 5. The calculation steps [3] are presented below and the results are given in Table 2.

### Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading (at $x = 10 \text{ m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 35/45</td>
<td>$h = 100.0 \text{ cm}$</td>
<td>$M_g = 1250 \text{ kNm}$</td>
</tr>
<tr>
<td>Y 1770</td>
<td>$b = 100.0 \text{ cm}$</td>
<td>$N_p = -3651.1 \text{ kN}$</td>
</tr>
<tr>
<td></td>
<td>$d = 95.0 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$L = 20.0 \text{ m}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_p = 28.5 \text{ cm}^2$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4: Simply Supported Beam
Table 2: Results

<table>
<thead>
<tr>
<th>Case</th>
<th>CS</th>
<th>Result</th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>$\sigma_{c,b}$ [MPa]</td>
<td>-12.47</td>
<td>-12.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_y$ [kNm]</td>
<td>-1435.91</td>
<td>-1435.91</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td></td>
<td>-4.82</td>
<td>-4.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_y$ [kNm]</td>
<td>-185.91</td>
<td>-185.91</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td></td>
<td>-11.76</td>
<td>-11.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_y$ [kNm]</td>
<td>-1406.11</td>
<td>-1406.11</td>
</tr>
<tr>
<td>IV</td>
<td>1</td>
<td></td>
<td>-4.51</td>
<td>-4.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_y$ [kNm]</td>
<td>-156.11</td>
<td>-156.11</td>
</tr>
</tbody>
</table>
4 Design Process

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:

Material:

Concrete: C 35/45

\[ E_{cm} = 34000 \text{ N/mm}^2 \]

Prestressing Steel: Y 1770

\[ E_p = 195000 \text{ N/mm}^2 \]

\[ f_{pk} = 1770 \text{ N/mm}^2 \]

\[ f_{p0.1k} = 1520 \text{ N/mm}^2 \]

Prestressing system: BBV L 19150 mm²

19 wires with area of 150 mm² each, giving a total of \( A_p = 28.5 \text{ cm}^2 \)

Cross-section:

\[ A_c = 1.0 \cdot 1.0 = 1 \text{ m}^2 \]

Diameter of duct \( \phi_{duct} = 97 \text{ mm} \)

Ratio \( \alpha_{E,p} = E_p / E_{cm} = 195000 / 34000 = 5.74 \)

\[ A_{c,netto} = A_c - \pi \cdot (\phi_{duct}/2)^2 = 0.9926 \text{ m}^2 \]

\[ A_{ideal} = A_c + A_p \cdot \alpha_{E,p} = 1.013 \text{ m}^2 \]

The force applied to a tendon, i.e. the force at the active end during tensioning, should not exceed the following value

\[ P_{max} = A_p \cdot \sigma_{p,max} \]

where \( \sigma_{p,max} = \min \{0.80f_{pk}; 0.90f_{p0.1k}\} \)

\[ P_{max} = A_p \cdot 0.80 \cdot f_{pk} = 28.5 \cdot 10^{-4} \cdot 0.80 \cdot 1770 = 4035.6 \text{ kN} \]

\[ P_{max} = A_p \cdot 0.90 \cdot f_{p0.1k} = 28.5 \cdot 10^{-4} \cdot 0.90 \cdot 1520 = 3898.8 \text{ kN} \]

\[ \rightarrow P_{max} = 3898.8 \text{ kN and } \sigma_{p,max} = 1368 \text{ N/mm}^2 \]

The value of the initial prestress force at time \( t = t_0 \) applied to the concrete immediately after tensioning and anchoring should not exceed the following value

\[ P_{m0}(x) = A_p \cdot \sigma_{p,m0}(x) \]

The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.3.6: Fig. 3.10, which can be seen in Fig 3.

The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.
where \( \sigma_{p,m0}(x) = \min \{ 0.75f_{pk} ; 0.85f_{p0,1k} \} \)

\[
P_{m0} = A_p \cdot 0.75 \cdot f_{pk} = 28.5 \cdot 10^{-4} \cdot 0.75 \cdot 1770 = 3783.4 \text{ kN}
\]

\[
P_{m0} = A_p \cdot 0.85 \cdot f_{p0,1k} = 28.5 \cdot 10^{-4} \cdot 0.85 \cdot 1520 = 3682.2 \text{ kN}
\]

\[\rightarrow P_{m0} = 3682.2 \text{ kN} \text{ and } \sigma_{p,m0} = 1292 \text{ N/mm}^2\]

Further calculations for the distribution of prestress forces and stresses along the beam are not in the scope of this Benchmark and will not be described here. The complete diagram can be seen in Fig. 5, after the consideration of losses at anchorage and due to friction, as calculated by SOFiSTiK. There the values of \( \sigma_{p,max} = 1368 \text{ N/mm}^2 \) and \( P_{m0} = 3682.2 \text{ kN} \) can be visualised.

Load Actions:

Self weight per length: \( \gamma = 25 \text{ kN/m} \)

\[\rightarrow g_1 = \gamma \cdot A = 25 \cdot 1 = 25 \text{ kN/m} \]

Safety factors at ultimate limit state

<table>
<thead>
<tr>
<th>Actions (unfavourable)</th>
<th>Safety factor at final state</th>
</tr>
</thead>
<tbody>
<tr>
<td>permanent</td>
<td>( \gamma_G = 1.35 )</td>
</tr>
<tr>
<td>prestress</td>
<td>( \gamma_P = 1.00 )</td>
</tr>
</tbody>
</table>

Combination coefficients at serviceability limit state

\( g_1 = 25 \text{ kN/m} \): for rare, frequent and quasi-permanent combination (for stresses)

At \( x = 10.0 \text{ m} \) middle of the span:

\[
M_g = g_1 \cdot L^2 / 8 = 1250 \text{ kNm}
\]

\[
N_p = P_{m0}(x = 10.0 \text{ m}) = -3653.0 \text{ kN} \text{ (from SOFiSTiK)}
\]

**Calculation of stresses** \( \sigma_{c,b} \) at \( x = 10.0 \text{ m} \) middle of the span:

Position of the tendon: \( z = 0, 3901 \text{ m} \)

- **Case I**: Prestress at construction stage section 0 (P cs0)

\[\rightarrow P_{m0,x=10} = -3653.0 \text{ kN} \]

\[N_p = -3653.0 \text{ kN}\]
\[ z_s = \text{the new position of the center of the cross-section for cs0} \]
\[ z_p = z + z_s \]

\[ M_p \text{ bending moment caused by pre-stressing} \]
\[ W_{1,cs0} \text{ cross-section moduli for construction stage 0 at the bottom left and right point} \]
\[ \sigma_{c,b} \text{ stress at the concrete at the bottom of the cross section} \]

\[ M_{p_1} = N_p \cdot z = -3653.0 \cdot 0.3901 = -1425.04 \text{ kNm} \]
\[ M_{p_2} = N_p \cdot z_s = -3653.0 \cdot 0.002978 = -10.879 \text{ kNm} \]
\[ M_p = -1425.04 - 10.879 = -1435.91 \text{ kNm} = M_y \]

\[ \sigma_{c,b} = \frac{N_p}{A_{c,net}} + \frac{M_y}{W_{1,cs0}} \]
\[ \sigma_{c,b} = -3653.0 \cdot 0.9926 + \frac{-1435.91}{0.1633} = -12.47 \text{ MPa} \]

**Case II:** Prestress and self-weight at con. stage sect. 0 (P+G cs0)

\[ N_p = -3653.0 \text{ kN and } M_g = 1250 \text{ kNm} \]
As computed above: \( M_p = -1435.91 \text{ kNm} \)
\[ M_y = 1250 - 1435.91 = -185.91 \text{ kNm} \]
\[ \sigma_{c,b} = -3653.0 \cdot 0.9926 + \frac{-185.91}{0.1633} = -4.82 \text{ MPa} \]

**Case III:** Prestress at con. stage sect. 1 (P cs1)

\[ N_p = -3653.0 \text{ kN and } M_{p_1} = -1425.04 \text{ kNm} \text{ (as above)} \]
\[ M_{p_2} = N_p \cdot z_s = -3653.0 \cdot (-0.00518) = 18.92 \text{ kNm} \]
\[ M_p = -1425.04 + 18.92 = -1406.11 \text{ kNm} = M_y \]
\[ \sigma_{c,b} = \frac{N_p}{A_{ideal}} + \frac{M_y}{W_{1,cs1}} \]
\[ \sigma_{c,b} = -3653.0 \cdot 1.013 + \frac{-1406.11}{0.172} = -11.76 \text{ MPa} \]

**Case IV:** Prestress and self-weight at con. stage sect. 1 (P+G cs1)

\[ N_p = -3653.0 \text{ kN and } M_g = 1250 \text{ kNm} \]
As computed above: \( M_p = -1406.11 \text{ kNm} \)
\[ M_y = 1250 - 1406.11 = -156.11 \text{ kNm} \]
\[ \sigma_{c,b} = \frac{N_p}{A_{ideal}} + \frac{M_y}{W_{1,cs1}} \]
\[ \sigma_{c,b} = -3653.0 \cdot 1.013 + \frac{-156.11}{0.172} = -4.51 \text{ MPa} \]
5 Conclusion

This example shows the calculation of the stresses, developed in the concrete cross-section due to prestress and bending. It has been shown that the results are reproduced with excellent accuracy.

6 Literature


