Benchmark Example No. 18

Creep and Shrinkage Calculation of a Rectangular Prestressed Concrete CS

SOFiSTiK | 2020
1 Problem Description

The problem consists of a simply supported beam with a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The time dependent losses are calculated, considering the reduction of stress caused by the deformation of concrete due to creep and shrinkage, under the permanent loads.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the calculation of creep and shrinkage on a prestressed concrete, subject to bending and prestress force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Creep and Shrinkage (Section 3.1.4)
- Annex B: Creep and Shrinkage (Section B.1, B.2)
- Time dependent losses of prestress for pre- and post-tensioning (Section 5.10.6)

The time dependant losses may be calculated by considering the following two reductions of stress [1]:

- due to the reduction of strain, caused by the deformation of concrete due to creep and shrinkage, under the permanent loads
- the reduction of stress in the steel due to the relaxation under tension.

In this Benchmark the stress loss due to creep and shrinkage will be examined.

3 Model and Results

Benchmark 17 is here extended for the case of creep and shrinkage developing on a prestressed concrete simply supported beam. The analysed system can be seen in Fig. 2, with properties as defined in Table 1. Further information about the tendon geometry and prestressing can be found in Benchmark 17. The beam consists of a rectangular cs and is prestressed and loaded with its own weight. A calculation of the creep and shrinkage is performed in the middle of the span with respect to DIN EN 1992-1-1:2004.
Creep and Shrinkage Calculation of a Rectangular Prestressed Concrete CS (German National Annex) [1], [2]. The calculation steps [3] are presented below and the results are given in Table 2 for the calculation with AQB. For CSM only the results of the creep coefficients and the final losses are given, since the calculation is performed in steps.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading (at x = 10 m)</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 35/45</td>
<td>$h = 100.0 , \text{cm}$</td>
<td>$M_g = 1250 , \text{kNm}$</td>
<td>$t_0 = 28 , \text{days}$</td>
</tr>
<tr>
<td>Y 1770</td>
<td>$b = 100.0 , \text{cm}$</td>
<td>$N_p = -3653.0 , \text{kN}$</td>
<td>$t_s = 0 , \text{days}$</td>
</tr>
<tr>
<td>RH = 80</td>
<td>$L = 20.0 , \text{m}$</td>
<td></td>
<td>$t_{\text{eff}} = 1000000 , \text{days}$</td>
</tr>
<tr>
<td></td>
<td>$A_p = 28.5 , \text{cm}^2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Simply Supported Beam

Table 2: Results

<table>
<thead>
<tr>
<th>Result</th>
<th>AQB</th>
<th>CSM+AQB</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{cs}$</td>
<td>$-18.85 \cdot 10^{-5}$</td>
<td>-</td>
<td>$-18.85 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$-31.58 \cdot 10^{-5}$</td>
<td>-</td>
<td>$-31.58 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>1.463</td>
<td>1.463</td>
<td>1.463</td>
</tr>
<tr>
<td>$\phi(t, t_0)$</td>
<td>1.393</td>
<td>1.393</td>
<td>1.393</td>
</tr>
<tr>
<td>$\Delta \sigma_{p,c+s}$ [MPa]</td>
<td>$-66.63$</td>
<td>$-67.30$</td>
<td>$-68.45$</td>
</tr>
<tr>
<td>$\Delta P_{c+s}$ [kN]</td>
<td>189.9</td>
<td>191.8</td>
<td>195.11</td>
</tr>
</tbody>
</table>
4 Design Process

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:

Material:
Concrete: C 35/45
\[ E_{cm} = 34077 \text{ N/mm}^2 \]
\[ f_{ck} = 35 \text{ N/mm}^2 \]
\[ f_{cm} = 43 \text{ N/mm}^2 \]

Prestressing Steel: Y 1770
\[ E_p = 195000 \text{ N/mm}^2 \]
\[ f_{pk} = 1770 \text{ N/mm}^2 \]

Prestressing system: BBV L19 150 mm²
19 wires with area of 150 mm² each, giving a total of \( A_p = 28.5 \text{ cm}^2 \)

Cross-section:
\[ A_c = 1.0 \cdot 1.0 = 1 \text{ m}^2 \]
Diameter of duct \( \phi_{duct} = 97 \text{ mm} \)
Ratio \( \alpha_{E,p} = E_p / E_{cm} = 195000 / 34077 = 5.7223 \)
\[ A_{c,netto} = A_c - \pi \cdot (\phi_{duct}/2)^2 = 0.9926 \text{ m}^2 \]
\[ A_{ideal} = A_c + A_p \cdot \alpha_{E,p} = 1.013 \text{ m}^2 \]

Load Actions:
Self weight per length: \( \gamma = 25 \text{ kN/m} \)
At \( x = 10.0 \text{ m} \) middle of the span:
\[ M_g = g_1 \cdot L^2 / 8 = 1250 \text{ kNm} \]
\[ N_p = P_{m0}(x = 10.0 \text{ m}) = -3653.0 \text{ kN} \] (from SOFiSTiK)

Calculation of stresses at \( x = 10.0 \text{ m} \) midspan:
Position of the tendon: \( z_{cp} = 0, 3901 \text{ m} \)

Prestress and self-weight at con. stage sect. 0 (P+G cs0)

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1The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.3.6: Fig. 3.10
2The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.
\[ N_p = -3653.0 \text{ kN and } M_g = 1250 \text{ kNm} \]

\[ M_{p1} = N_p \cdot z_{cp} = -3653.0 \cdot 0.3901 = -1425.04 \text{ kNm} \]

\[ M_{p2} = N_p \cdot z_s = -3653.0 \cdot 0.002978 = -10.879 \text{ kNm} \]

\[ M_p = 1425.04 - 10.879 = 1435.91 \text{ kNm} \]

\[ M_y = 1250 - 1435.91 = -185.91 \text{ kNm} \]

\[ \sigma_{c,OP} = \frac{-3653.0}{0.9926} + \frac{-185.91}{0.1633} = -4.82 \text{ MPa} \]

**Calculation of creep and shrinkage** at \( x = 10.0 \text{ m} \) midspan:

\( t_0 = 28 \text{ days} \)

\( t_s = 0 \text{ days} \)

\( t = t_{eff} + t_0 = 1000000 + 28 = 1000028 \text{ days} \)

\[ \epsilon_{cs} = \epsilon_{cd} + \epsilon_{ca} \]

\[ \epsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \epsilon_{cd,0} \]

The development of the drying shrinkage strain in time is strongly depends on \( \beta_{ds}(t, t_s) \) factor. SOFiSTiK accounts not only for the age at start of drying \( t_s \) but also for the influence of the age of the prestressing \( t_0 \). Therefore, the calculation of factor \( \beta_{ds} \) reads:

\[ \beta_{ds} = \beta_{ds}(t, t_s) - \beta_{ds}(t_0, t_s) \]

\[ \beta_{ds} = \frac{(t - t_s)}{(t - t_s) + 0.04 \sqrt{h_0^3}} - \frac{(t_0 - t_s)}{(t_0 - t_s) + 0.04 \sqrt{h_0^3}} \]

\[ \beta_{ds} = \frac{(1000028 - 0)}{(1000028 - 0) + 0.04 \sqrt{500^3}} - \frac{(28 - 0)}{(28 - 0) + 0.04 \sqrt{500^3}} \]

\[ \beta_{ds} = 0.99955 - 0.05892 = 0.94063 \]

\[ k_h = 0.70 \text{ for } h_0 \geq 500 \text{ mm} \]

\[ \epsilon_{cd,0} = 0.85 \left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp \left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right)\right] \cdot 10^{-6} \cdot \beta_{RH} \]

\[ \beta_{RH} = 1.55 \left[1 - \left(\frac{RH}{RH_0}\right)^3\right] = 1.55 \left[1 - \left(\frac{80}{100}\right)^3\right] = 0.7564 \]
\[
\varepsilon_{cd,0} = 0.85 \left[ (220 + 110 \cdot 4) \cdot \exp \left( -0.12 \cdot \frac{43}{10} \right) \right] \cdot 10^{-6} = 0.7564 \\
\varepsilon_{cd,0} = 2.533 \cdot 10^{-4} \\
\varepsilon_{cd} = 0.94063 \cdot 0.70 \cdot 2.533 \cdot 10^{-4} = 0.0001668 \\
\varepsilon_{cd} = 1.668 \cdot 10^{-4} = 0.1668 \text{ } ^{\circ}/_{\infty} \\
\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) \\
\varepsilon_{ca}(\infty) = 2.5 \left( f_{ck} - 10 \right) \cdot 10^{-6} = 2.5 \left( 35 - 10 \right) \cdot 10^{-6} \\
\varepsilon_{ca}(\infty) = 6.25 \cdot 10^{-5} = 0.0625 \text{ } ^{\circ}/_{\infty} \\
\text{Proportionally to } \beta_{ds}(t, t_5), \text{ SOFiSTiK calculates factor } \beta_{as} \text{ as follows:} \\
\beta_{as} = \beta_{as}(t) - \beta_{as}(t_0) \\
\beta_{as} = 1 - e^{-0.2 \cdot \sqrt{\varepsilon}} - \left( 1 - e^{-0.2 \cdot \sqrt{t_0}} \right) = e^{-0.2 \cdot \sqrt{t_0}} - e^{-0.2 \cdot \sqrt{\varepsilon}} \\
\beta_{as} = 0.347 \\
\varepsilon = \varepsilon_{cd,0} + \varepsilon_{ca}(\infty) = 2.533 \cdot 10^{-4} + 6.25 \cdot 10^{-5} \\
\varepsilon = -35.18 \cdot 10^{-5} \\
\varepsilon_{ca} = 0.347 \cdot 6.25 \cdot 10^{-5} = 2.169 \cdot 10^{-5} = 0.02169 \text{ } ^{\circ}/_{\infty} \\
\varepsilon_{cs} = 1.668 \cdot 10^{-4} + 2.169 \cdot 10^{-5} = -18.85 \cdot 10^{-5} \\
\phi(t, t_0) = \phi_0 \cdot \beta_c(t, t_0) \\
\phi_0 = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \\
\phi_{RH} = \left[ 1 + \frac{1 - RH/100}{0.1 \cdot \sqrt{\rho_0}} \cdot \alpha_1 \right] \cdot \alpha_2 \\
\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = 16.8/\sqrt{43} = 2.562 \\
\alpha_1 = \left[ \frac{35}{f_{cm}} \right]^{0.7} \leq 1 = 0.8658 \\
\alpha_2 = \left[ \frac{35}{f_{cm}} \right]^{0.2} \leq 1 = 0.9597 \\
\alpha_3 = \left[ \frac{35}{f_{cm}} \right]^{0.5} \leq 1 = 0.9022 \\
\phi_{RH} = \left[ 1 + \frac{1 - 80/100}{0.1 \cdot \sqrt{500}} \cdot 0.8658 \right] \cdot 0.9597 = 1.1691 \\
\beta(t_0) = \frac{1}{0.1 + t_0^{0.20}} \\
\text{Annex B.2 (1): } \alpha_{ds1}, \alpha_{ds2} \text{ coefficients depending on type of cement.} \\
\text{For class N } \alpha_{ds1} = 4, \alpha_{ds2} = 0.12 \\
\text{3.1.4 (6): Eq. 3.11: } \varepsilon_{ca} \text{ autogenous shrinkage strain} \\
\text{3.1.4 (6): Eq. 3.12: } \varepsilon_{ca}(\infty) \\
\text{3.1.4 (6): Eq. 3.13: } \beta_{as} \\
\text{3.1.4 (6): Eq. 3.11: } \epsilon \text{ absolute shrinkage strain} \\
\text{negative sign to declare losses} \\
\text{negative sign to declare losses} \\
\text{Annex B.1 (1): Eq. B.1: } \phi(t, t_0) \text{ creep coefficient} \\
\text{Annex B.1 (1): Eq. B.2: } \phi_0 \text{ notional creep coefficient} \\
\text{Annex B.1 (1): Eq. B.3: } \phi_{RH} \text{ factor for effect of relative humidity on creep} \\
\text{Annex B.1 (1): Eq. B.4: } \beta(f_{cm}) \text{ factor for effect of concrete strength on creep} \\
\text{Annex B.1 (1): Eq. B.5: } \beta(t_0) \text{ factor for effect of concrete age at loading on creep} \\
\text{Annex B.1 (1): Eq. B.6: } \alpha_1, \alpha_2, \alpha_3 \text{ coefficients to consider influence of concrete strength}
t_0 = t_{0,T} \left( \frac{9}{2 + t_{0,T}^{1/2}} + 1 \right)^\alpha \geq 0.5

t_T = \sum_{i=1}^{\infty} e^{-\left(4000/[273+T(\Delta t_i)]-13.65\right)} \cdot \Delta t_i

t_{0,T} = 28 \cdot e^{-\left(4000/[273+20]-13.65\right)} = 28 \cdot 1.0 = 28.0

\Rightarrow t_0 = 28.0 \cdot \left( \frac{9}{2 + 28.0^{1/2}} + 1 \right)^0 = 28.0

\beta(t_0) = \frac{1}{(0.1 + 28.0^{0.20})} = 0.48844

\beta_c(t, t_0) = \left[ \frac{(t - t_0)}{(\beta_H + t - t_0)} \right]^{0.3}

\beta_H = 1.5 \cdot [1 + (0.012 \cdot RH)^{18}] \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3

\beta_H = 1.5 \cdot [1 + (0.012 \cdot 80)^{18}] \cdot 500 + 250 \cdot 0.9022

\beta_H = 1335.25 \leq 1500 \cdot 0.9022 = 1353.30

\Rightarrow \beta_c(t, t_0) = 0.9996

\phi_0 = 1.1691 \cdot 2.562 \cdot 0.48844 = 1.463

\phi(t, t_0) = 1.463 \cdot 0.9996/1.05 = 1.393

According to EN, the creep value is related to the tangent Young’s modulus E_c, where E_c being defined as 1.05 \cdot E_{cm}. To account for this, SOFiSTiK adopts this scaling for the computed creep coefficient (in SOFiSTiK, all computations are consistently based on E_{cm}).

\Delta P_{c+s+r} = A_p \cdot \Delta \sigma_{p,c+s+r} = A_p \cdot \frac{E_p}{E_{cm}} \cdot \frac{A_p}{A_c} \cdot \frac{A_c}{A_{cp}} \cdot \frac{E_{cm}}{E_{cp}} \cdot \phi(t, t_0) \cdot \sigma_{c,QP}

\Delta P_{c+s+r} = \frac{\epsilon_{cs} \cdot E_p + 0.8 \Delta \sigma_{pr} + E_{cp} \phi(t, t_0) \cdot \sigma_{c,QP}}{1 + \frac{E_p}{E_{cm}} \cdot \frac{A_p}{A_c} \cdot \left(1 + \frac{A_c}{I_c} \cdot \frac{z_{cp}^2}{2}ight) \cdot [1 + 0.8 \phi(t, t_0)]}

In this example only the losses due to creep and shrinkage are taken into account, the reduction of stress due to relaxation (\Delta \sigma_{pr}) is ignored.

\Delta \sigma_{p,c+s} = \frac{-0.1885 \cdot 10^{-3} \cdot 195000 + 5.7223 \cdot 1.393 \cdot (-4.82)}{1 + 5.7223 \cdot 28.5 \cdot 10^{-4} \cdot 0.9926 \cdot 0.3901^2} \cdot [1 + 0.8 \cdot 1.393]

\Delta \sigma_{p,c+s} = \frac{-68.46 \text{ MPa}}{1}

\Delta P_{c+s} = A_p \cdot \Delta \sigma_{p,c+s} = 28.5 \cdot 10^{-4} \cdot 68.46 \cdot 103 = 195.11 \text{ kN}
5 Conclusion

This example shows the calculation of the time dependent losses due to creep and shrinkage. It has been shown that the results are in very good agreement with the reference solution.

6 Literature

