Benchmark Example No. 25

Shear between web and flanges of Hollow CS acc. DIN EN 1992-2
1 Problem Description

The problem consists of a Hollow section, as shown in Fig. 1. The CS is designed for shear, the shear between web and flanges of the Hollow CS is considered and the required reinforcement is determined.

2 Reference Solution

This example is concerned with the shear design of Hollow-sections, for the ultimate limit state. The content of this problem is covered by the following parts of DIN EN 1992-2:2010 [1] and DIN EN 1992-1-1:2004 [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 2 and as defined in DIN EN 1992-1-1:2004 [2] (Section 3.2.7).
3 Model and Results

The Hollow-section, with properties as defined in Table 1, is to be designed for shear, with respect to DIN EN 1992-2:2010 (German National Annex) [1]. The structure analysed, consists of a single span beam with a distributed load in gravity direction. The cross-section geometry, as well as the shear cut under consideration can be seen in Fig. 3.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 30/37</td>
<td>$h_n = 35 \text{ cm}$, $h = 75.0 \text{ cm}$</td>
<td>$P_g = 155 \text{ kN/m}$</td>
</tr>
<tr>
<td>B 500B</td>
<td>$h_f = 20 \text{ cm}$</td>
<td>$M_t = 100 \text{ kN/m}$</td>
</tr>
<tr>
<td></td>
<td>$d_1 = 5.0 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b_n = 110 \text{ cm}$, $b = 160 \text{ cm}$</td>
<td></td>
</tr>
</tbody>
</table>
The system with its loading as well as the moment and shear force are shown in Fig. 4-7. The reference calculation steps are presented in the next section and the results are given in Table 2.

**Figure 4: Loaded Structure (P_g)**

**Figure 5: Resulting Bending Moment M_y**

**Figure 6: Resulting Torsional Moment M_t**

**Figure 7: Resulting Shear Force V_z**

**Table 2: Results**

<table>
<thead>
<tr>
<th>At beam 1001</th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{s1} [cm^2] ) at ( x = 1.0 ) m</td>
<td>22.49</td>
<td>22.52</td>
</tr>
<tr>
<td>( V_{Rd,c} [kN] )</td>
<td>91.91</td>
<td>91.91</td>
</tr>
</tbody>
</table>
Table 2: (continued)

<table>
<thead>
<tr>
<th>At beam 1001</th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{Rd,max}$ [kN]</td>
<td>1098.43</td>
<td>1098.461</td>
</tr>
<tr>
<td>$\cot \theta$</td>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td>$z$ [cm] at $x = 1.0 , m$</td>
<td>63.00</td>
<td>67.00</td>
</tr>
<tr>
<td>$V_{Ed} = \Delta F_d$ [kN]</td>
<td>350.13</td>
<td>352.35</td>
</tr>
<tr>
<td>$a_{sf,l}$ [$cm^2$]</td>
<td>8.23</td>
<td>8.42</td>
</tr>
<tr>
<td>$a_{sf,r}$ [$cm^2$]</td>
<td>0.81</td>
<td>0.83</td>
</tr>
</tbody>
</table>
4 Design Process\(^1\)

Design with respect to DIN EN 1992-2:2010 (NA) \([1]\):\(^2\)

Material:

Concrete: \(\gamma_c = 1.50\)

Steel: \(\gamma_s = 1.15\)

\(f_{ck} = 30 \text{ MPa}\)

\(f_{cd} = a_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 30 / 1.5 = 17 \text{ MPa}\)

\(f_{yk} = 500 \text{ MPa}\)

\(f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78 \text{ MPa}\)

\(\sigma_{sd} = 456.52 \text{ MPa}\)

Design loads

- Design Load for beam 1001, \(x=0.0\) m:
  
  \[M_{Ed,x=0.0} = 0.0 kNm\]

- Design Load for beam 1001, \(x=1.0\) m:
  
  \[M_{Ed,x=1.0} = 697.5 kNm\]

Calculating the longitudinal reinforcement:

- For beam 1001, \(x=0.0\) m
  
  \[\mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{0.0 \cdot 10^{-3}}{1.60 \cdot 0.70^2 \cdot 17.00} = 0.00\]

- For beam 1001, \(x=1.0\) m
  
  \[\mu_{Eds} = \frac{M_{Eds}}{b_{eff} \cdot d^2 \cdot f_{cd}} = \frac{697.5 \cdot 10^{-3}}{1.60 \cdot 0.70^2 \cdot 17.00} = 0.0523\]

\(\omega \approx 0.053973, \xi \approx 0.9658, \xi = 0.07833\) (interpolated)

\[A_{s1} = \frac{1}{\sigma_{sd}} \cdot (\omega \cdot b \cdot d \cdot f_{cd} + N_{Ed})\]

\[A_{s1} = \frac{1}{456.52} \cdot (0.053973 \cdot 1.6 \cdot 0.7 \cdot 17.0) \cdot 100^2 = 22.52 \text{ cm}^2\]

\[z = \xi \cdot d = 0.9658 \cdot 0.70 \text{ m} \approx 67.00 \text{ cm}\]

Calculating the shear between flange and web

The shear force, is determined by the change of the longitudinal force, at the junction between one side of a flange and the web, in the separated flange:

- The tools used in the design process are based on steel stress-strain diagrams, as defined in \([2]\) 3.2.7.(2), Fig. 3.8, which can be seen in Fig. 2.

\(^1\) The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) \([1]\), unless otherwise specified.
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\[ x = 0.0738 \cdot d = 0.07833 \cdot 70 = 5.48 \, \text{cm} < h_f = 20 \, \text{cm} \]

\[ \Delta F_d = \left( \frac{M_{Ed,x=1.0}}{z} - \frac{M_{Ed,x=0.0}}{z} \right) \cdot \frac{h_f \cdot b}{h_f \cdot b_h} \]

For beam 1001 (x=0.00 m) \( M_{Ed} = 0.00 \) therefore:

\[ \Delta F_d = \left( \frac{697.5}{0.68047} - 0 \right) \cdot \frac{1.1}{1.6} = 352.35 \cdot 2 = 704.70 \, \text{kN} \]

The longitudinal shear stress \( V_{Ed,V} \) at the junction between one side of a flange and the web is determined by the change of the normal (longitudinal) force in the part of the flange considered, according to:

\[ V_{Ed,V} = \frac{2}{h_f \cdot \Delta x} \]

In our case \( \Delta x = 1.0 \) because the beam length is \( 1.00 \, \text{m} \).

Please note that AQB is outputting the results per length.

\[ V_{Ed,V} = \frac{352.35}{20 \cdot 100} = 0.176 \, \text{kN/m}^2 = 1.76 \, \text{MPa} \]

Checking the maximum \( V_{Rd,max} \) value to prevent crushing of the struts in the flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied:

\[ V_{Ed,V} \leq V_{Rd,max} = V \cdot f_{cd} \cdot \sin \theta_f \cdot \cos \theta_f \]

\[ V_{Rd,max} = V \cdot f_{cd} \cdot \sin \theta_f \cdot \cos \theta_f \]

According to DIN EN 1992-2, NDP 6.2.4:

\[ V = V_1 \]

\[ V_1 = 0.75 \cdot V_2 \]

\[ V_2 = 1.1 - \frac{f_{ck}}{500} \leq 1.0 \]

\[ V_2 = 1.1 - \frac{30}{500} = 1.1 - 0.06 = 1.04 \geq 1.0 \] \( \rightarrow V_2 = 1.0 \)

\[ V_1 = 0.75 \cdot 1.0 = 0.75 \] \( \rightarrow V = 0.75 \)

The \( \theta \) value is calculated:

\[ V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left( 1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}} \right) \cdot b_w \cdot z \]

\( b_w \rightarrow h_f, \quad z \rightarrow \Delta x, \quad c = 0.5 \)

\[ V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left( 1 - 1.2 \cdot \frac{\sigma_{cd}}{f_{cd}} \right) \cdot h_f \cdot \Delta x \]

\[ V_{Rd,cc} = 0.5 \cdot 0.48 \cdot 30^{1/3} \cdot \left( 1 - 1.2 \cdot \frac{0}{17.00} \right) \cdot 0.20 \cdot 1.0 \]
\[ V_{Rd,cc} = 0.14914 \text{ MN} = 149.1471 \text{ kN} \]

\[ 1.0 \leq \cot \theta \leq \frac{1.2 + 1.4 \cdot \Delta \sigma_{cd}/f_{cd}}{1 - \frac{V_{Rd,cc}}{V_{Ed}}} \leq 1.75 \]

Because \( M_T \neq 0.0 \rightarrow \tau_T \neq 0.0 \):

\[ V_{Rd,cc} = \frac{V_{Rd,cc}}{h_f \cdot \Delta x} = \frac{149.1471}{20 \cdot 100} = 0.0745 \text{ kN/cm}^2 = 0.745 \text{ MPa} \]

The internal forces must be in equilibrium, see Fig. 8!!! If the internal forces are not in equilibrium state, then AQB will print erroneous results. Therefore the internal forces must be taken from the system (calculated by using e.g. ASE or STAR2).

\[ \cot \theta = \frac{1.2}{1 - 0.745/1.76} = 2.0807 \geq 1.75 \rightarrow 1.75 \]

AQB is checking and iterating if the \( \cot \theta \) is less than (<) the maximum \( \cot \theta \) defined in the norm. If yes, then this value will be taken by AQB.

\[ \tan \theta = \frac{1}{\cot \theta} = \frac{1}{1.75} = 0.5714 \rightarrow \theta = 29.74488^\circ \]

\[ V_{Rd,max} = 0.75 \cdot 17.00 \cdot \sin 29.7448 \cdot \cos 29.7448 = 5.492 \text{ MPa} \]

\[ V_{Rd,max} = V_{Rd,max} \cdot h_f \cdot \Delta x = 5.492 \cdot 0.20 \cdot 1.0 = 1.098 \text{ MN} \]

\[ V_{Rd,max} = 1098.461 \text{ kN} \]

**Checking the value** \( V_{Rd,c} \)

If \( V_{Ed} \) is less than or equal to \( V_{Rd,c} = k \cdot f_{ctd} \) no extra reinforcement above that for flexure is required.

\[ V_{Rd,c} = k \cdot f_{ctd} \]

For concrete C 30/37 \( \rightarrow f_{ctd} = 1.15 \text{ MPa} \)

\[ V_{Rd,c} = 0.4 \cdot 1.15 = 0.46 \text{ MPa} \]

\[ V_{Rd,c} = V_{Rd,c} \cdot h_f \cdot \Delta x = 0.046 \cdot 20 \cdot 100 \approx 91.91 \text{ kN} \]

- Calculating the necessary transverse reinforcement (\( \tau_v \) only):
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\[ \alpha_{sf,V} = \frac{\nu_{Ed} \cdot h_f}{\cot \theta_f \cdot f_{yd}} \]

\[ \alpha_{sf,V} = \frac{1.76 \cdot 0.20}{1.75 \cdot 434.78} \cdot 100^2 = 4.63 \text{ cm}^2 \]

- Calculating the necessary torsional reinforcement:

**HINT:** Please note that in AQB the torsional stress **IS NOT** calculated by using the formula:

\[ \tau_t = \frac{M_t}{2 \cdot A_k \cdot t_{eff}} \]

AQB is calculating the \( \tau_t \) stresses by multiplying \( M_t \) with the torsional resistance \( 1/W_t \) (from AQUA - calculated by using FEM).

\[ \tau_t = M_t \cdot \frac{1}{W_t} \]

From AQUA the torsional resistance value \( 1/W_t \) is interpolated and \( 1/W_t = 2.8841 \text{ 1/m}^3 \). To see the values \( 1/W_t \) input ECHO FULL EXTR in AQUA and see table "Construction and Selected Result Points".

\[ \tau_t = 2.8841 \cdot 500 \cdot 10^{-3} = 1.44 \text{ MPa} \]

\[ \alpha_{sf,T} = \frac{\tau_t \cdot t_{eff}}{\cot \theta_f \cdot f_{yd}} = \frac{1.44 \cdot 0.20}{1.75 \cdot 434.78} \cdot 100^2 = 3.79 \text{ cm}^2 \]

- Total reinforcement:

\[ \alpha_{sf,left} = \alpha_{sf,V} + \alpha_{sf,T} = 4.63 + 3.79 = 8.42 \text{ cm}^2 \]

\[ \alpha_{sf,right} = |\alpha_{sf,V} - \alpha_{sf,T}| = |4.63 - 3.79| = 0.83 \text{ cm}^2 \]
5 Conclusion

This example is concerned with the calculation of the shear between web and flanges of a Hollow CS. It shows partially the work-flow how AQB calculates the shear between web and flanges.

Please note that it is very difficult to show all steps how AQB calculates internally the \( \tau \) and \( \sigma \) stresses, therefore some steps are skipped. The reference example is just an approximation that shows the results by using hand-calculation. It has been shown that the results calculated by using hand-calculation and the AQB module are reproduced with very good accuracy.

The \( \tau_t \) and \( \tau_v \) values between cross-section points are interpolated and calculated by using Finite Element Method.

![Figure 9: The \( \tau_t \) stresses (vector) calculated by using Finite Element Method](image)

![Figure 10: The \( \tau_t \) stresses (fill) calculated by using Finite Element Method](image)
6 Literature

