1  Problem Description

The problem consists of a simply supported beam with a steel HEA 200 section which is restrained in the middle of the length. The column is subjected to compression and bending as shown in Fig. 1.

![Figure 1: Problem Description](image)

2  Reference Solution

This example is concerned with the buckling resistance of steel members. It deals with the spatial behavior of the beam and the occurrence of lateral torsional buckling as a potential mode of failure. The content of this problem is covered by the following parts of DIN EN 1993-1-1:2005 [1]:

- Structural steel (Section 3.2)
• Classification of cross-sections (Section 5.5)
• Buckling resistance of members (Section 6.3)
• Method 2: Interaction factors $k_{ij}$ for interaction formula in 6.3.3(4) (Annex B)

3 Model and Results

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Cross-Section Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S 235$</td>
<td>$h = 190 , mm$</td>
<td>$H = 8.0 , m$</td>
<td>$q = 4.00 , kN/m$</td>
</tr>
<tr>
<td>$E = 210000 , N/mm^2$</td>
<td>$b = 200 , mm$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_y = 235 , N/mm^2$</td>
<td>$c = 32 , mm$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 0.3$</td>
<td>$r = 18 , mm$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G = 81000 , N/mm^2$</td>
<td>$t_f = 10 , mm$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{M0} = 1.0$</td>
<td>$t_w = 6.5 , mm$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{M1} = 1.1$</td>
<td>$A = 5308 , mm^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$i_y = 82.8 , mm$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$i_z = 49.8 , mm$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_y = 3690 , cm^4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_z = 1340 , cm^4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I_t = 21 , cm^4$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th></th>
<th>SOF.</th>
<th>Ref. [2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{cr,y} , [m]$</td>
<td>8.00</td>
<td>8.00</td>
</tr>
<tr>
<td>$L_{cr,z} , [m]$</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>$N_{pl,Rd} , [kN]$</td>
<td>1265.4</td>
<td>1264.3</td>
</tr>
<tr>
<td>$M_{pl,y,Rd} , [kNm]$</td>
<td>100.96</td>
<td>100.90</td>
</tr>
<tr>
<td>$\bar{\lambda}_y$</td>
<td>1.029</td>
<td>1.029</td>
</tr>
<tr>
<td>$\bar{\lambda}_z$</td>
<td>0.855</td>
<td>0.855</td>
</tr>
<tr>
<td>$N_{cr,z} , [kN]$</td>
<td>1730.1</td>
<td>1736</td>
</tr>
<tr>
<td>$M_{cr} , [kNm]$</td>
<td>245.47</td>
<td>220.9</td>
</tr>
</tbody>
</table>
Table 2: (continued)

<table>
<thead>
<tr>
<th></th>
<th>SOF.</th>
<th>Ref. [2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{LT}$</td>
<td>0.641</td>
<td>0.676</td>
</tr>
<tr>
<td>$k_{yy}$</td>
<td>1.292</td>
<td>1.304</td>
</tr>
<tr>
<td>$k_{zy}$</td>
<td>0.935</td>
<td>0.936</td>
</tr>
</tbody>
</table>
4 Design Process

Design Loads:
\[ N_d = 300 \, kN \]
\[ q = 4.0 \, kN/m \]
\[ M_{y,d} = q \cdot L^2/8 = 4.0 \cdot 8^2/8 = 32.0 \, kNm \]
\[ M_{z,d} = 0 \]

Buckling lengths:
\[ L_{cr,y} = 8.00 \, m \]
\[ L_{cr,z} = 4.00 \, m \]
\[ \xi = 1.35 \]

Characteristic values:
\[ N_{Rk} = N_{pl,Rd} = 1264.3 \, kN \]
\[ M_{y,Rk} = M_{pl,y,Rd} = 100.9 \, kNm \]
\[ I_x = 1340 \, cm^4 \]
\[ I_w = 108000 \, cm^6 \]
\[ I_t = 21.0 \, cm^4 \]
\[ i_y = 8.28 \, cm \]
\[ i_x = 4.98 \, cm \]
\[ z_p = -9.5 \, cm \]

Buckling around the y-y axis:
\[ \bar{x}_y = 800/(8.28 \cdot 93.9) = 1.029 \rightarrow \chi = 0.58 \text{ (Curve b)} \]
\[ \bar{x}_z = 400/(4.98 \cdot 93.9) = 0.855 \rightarrow \chi = 0.63 \text{ (Curve c)} \]

Critical loading:
\[ N_{cr,z} = \pi^2 \cdot 21000 \cdot 1340/400^2 = 1736 \, kN \]
\[ c^2 = (108000 + 0.039 \cdot 400^2 \cdot 21.0)/1340 = 178.4 \, cm^2 \]
\[ M_{cr} = 1.35 \cdot 1736 \cdot \left[ \sqrt{178.4 + 0.25 \cdot 9.5^2 + 0.5 \cdot (-9.5)} \right] \cdot 10^2 \]
\[ M_{cr} = 220.9 \, kNm \]

\footnote{The sections mentioned in the margins refer to DIN EN 1993-1-1:2005 [1] unless otherwise specified.}
Figure 2: Internal forces $M_y$ and $N$

\[ \bar{\lambda}_{LT} = \sqrt{\frac{100.9}{220.9}} = 0.676 \]

$h/b = 190/200 = 0.95 < 2.0 \rightarrow \text{Class b}$

\[ \chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \cdot \bar{\lambda}_{LT}^2}} \]

\[ \Phi_{LT} = 0.5 \cdot \left[ 1 + \alpha_{LT} \cdot (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \cdot \bar{\lambda}_{LT}^2 \right] \]

\[ \chi_{LT} = 0.88 < 2.188 = 1/0.676^2 \]

\[ M_{b,Rd} = M_{pl,y,Rd} \cdot \frac{\chi_{LT}}{f} \cdot \frac{1}{\gamma_{M1}} \]

\[ f = 1 - 0.5 \cdot (1 - k_L) \cdot \left[ 1 - 2.0 \cdot (\bar{\lambda}_{LT} - 0.8^2) \right] \]

\[ f = 1 - 0.5 \cdot (1 - 0.94) \cdot \left[ 1 - 2.0 \cdot (0.676 - 0.8^2) \right] \]

\[ f = 0.972 \]

\[ M_{b,Rd} = 100.9 \cdot 0.88 \cdot 1.1 \cdot 0.972 \]

\[ M_{b,Rd} = 83.045 \, kNm \]

**Equivalent uniform moment factors:**

$\bar{\lambda}_{LT}$ - non dimensional slenderness for lateral torsional buckling

$\chi_{LT}$ - reduction factor

$f$ - the value $f$ may be defined in National Annex, see EC 3, §6.3.2.3(2)

$m_{b,Rd}$ - design buckling resistance moment - EC 3, §6.3.2.1, Eq. 6.55
for $L_{cr,y} = 8.00 \text{ m}$:

- $\alpha_h = \frac{M_h}{M_s} = 0.00$
- $\psi = 1.00$
- $c_{my} = 0.95 + 0.05 \cdot 0.00 = 0.95$

for $L_{cr,z} = 4.00 \text{ m}$:

- $\alpha_h = \frac{M_h}{M_s} = 0.75$
- $\psi = 0.00$
- $c_{mLT} = 0.2 + 0.8 \cdot 0.75 = 0.80$

**Figure 3: Calculating the equivalent uniform moment factors**

**Interaction factors:**

$k_{xy}, k_{zy}$ - Annex B: Method 2, interaction factors for interaction formula in §6.3.3(4)
\[ k_{yy} = 0.95 \left(1 + 1.029 - 0.2\right) \cdot 300 \cdot \frac{1.1}{0.580 \cdot 1264.3} \]
\[ = 1.304 \]
\[ \leq 0.95 \left(1 + 0.8 \cdot 300 \cdot \frac{1.1}{0.580 \cdot 1264.3}\right) = 1.292 \]
\[ \leq 1.80 \]

\[ k_{zy} = \left[1 - \frac{0.1 \cdot 0.855}{0.8 - 0.25} \cdot \frac{300 \cdot 1.1}{0.63 \cdot 1264.3}\right] = 0.936 \]
\[ \geq \left[1 - \frac{0.1}{0.8 - 0.25} \cdot \frac{300 \cdot 1.1}{0.63 \cdot 1264.3}\right] = 0.925 \]
\[ \geq 0.3 \cdot 300 \cdot \frac{1.1}{0.63 \cdot 1264.3} = 0.138 \]

**Lateral Torsional Buckling check:**

\[ \frac{300}{0.58 \cdot 1264.3/1.1} + 1.292 \cdot \frac{32.0 + 0.0}{0.88 \cdot 100.9/1.1} + 0.0 = 0.96 \leq 1.0 \]

\[ \frac{300}{0.63 \cdot 1264.3/1.1} + 0.936 \cdot \frac{32.0 + 0.0}{0.88 \cdot 100.9/1.1} + 0.0 = 0.79 \leq 1.0 \]
5 Conclusion

This example shows the check for lateral torsional buckling of steel members. The small deviations that occur in some results come from the fact that there are some small differences in the sectional values and elastic critical loadings ($M_{cr}$, $N_{cr}$). Therefore, these deviations are of no interest for the specific verification process. In conclusion, it has been shown that the results are reproduced with excellent accuracy.

6 Literature
