Benchmark Example No. 33

Designing a Reinforced Concrete Column acc. DIN EN 1992-1-1

SOFiSTiK | 2020
Overview

<table>
<thead>
<tr>
<th>Design Code Family(s):</th>
<th>DIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Code(s):</td>
<td>DIN EN 1992-1-1</td>
</tr>
<tr>
<td>Module(s):</td>
<td>COLUMN</td>
</tr>
<tr>
<td>Input file(s):</td>
<td>column_din_en_1992_nkv.dat</td>
</tr>
</tbody>
</table>

1 Problem Description

The problem consists of a reinforced concrete column positioned at edge of the building, as shown in Fig. 1. Different loading conditions and the design approach by using the *nominal curvature* method are examined.

The main goal of this benchmark is to verify and compare the SOFiSTiK results with the reference example *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau* [1, Example 10]. In this example the Ultimate Limit State (ULS) of the prefabricated column is verified.

![Figure 1: Column D1](image)

2 Reference Solution

This example is concerned with the design of reinforced concrete column. The content of this problem is covered by following parts of DIN EN 1992-1-1:2004 [2]:

- Concrete cover (Section 4.4.1.1 and (NDP) Tab. 4.4DE)
- Materials (Section 3.1 and 3.2)
- Analysis of second order effects with axial load (Section 5.8)
- Geometric imperfections (Section 5.2)
- Columns (Section 9.5)
- Shear (Section 6.2.2)
The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as defined in DIN EN 1992-1-1:2004 [2] (Section 3.2.7).

### 3 Model and Results

The column, with properties as defined in Table 1, is to be designed for ultimate limit state, with respect to DIN EN 1992-1-1:2004 (German National Annex) [2] [3]. The calculation steps with loading conditions are presented below and the results are given in Table 3.

### Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 30/37</td>
<td>$b_1/h_1 = 45/40 \text{ cm}$</td>
<td>$G_k = 431.0 \text{ kN}$</td>
</tr>
<tr>
<td>B 500B</td>
<td>$b_2/h_2 = 17/40 \text{ cm}$</td>
<td>$Q_{k,s} = 68.0 \text{ kN}$</td>
</tr>
<tr>
<td></td>
<td>$d_y = 41 \text{ cm}$</td>
<td>$q_{k,w1} = +4.32 \text{ kN/m}$</td>
</tr>
<tr>
<td></td>
<td>$h = 6.20 + 1.9 = 8.1 \text{ m}$</td>
<td>$q_{k,w2} = -1.85 \text{ kN/m}$</td>
</tr>
</tbody>
</table>
Table 2: Results

<table>
<thead>
<tr>
<th>Units</th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$ [kN]</td>
<td>-633.0</td>
<td>-633.0</td>
</tr>
<tr>
<td>$M_y$ [kNm]</td>
<td>100.28</td>
<td>100.0</td>
</tr>
<tr>
<td>$M_z$ [kNm]</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$e_0$ [mm]</td>
<td>158.4$^1$</td>
<td>179.0</td>
</tr>
<tr>
<td>$e_1$ [mm]</td>
<td>20.7</td>
<td>21.0</td>
</tr>
<tr>
<td>$A_{s,req,1}$ [cm$^2$]</td>
<td>15.45$^1$</td>
<td>16.20</td>
</tr>
<tr>
<td>$A_{s,req,2}$ [cm$^2$]</td>
<td>4.52</td>
<td>4.52</td>
</tr>
</tbody>
</table>

$^1$See conclusion (Section 5) for more details why the results are different compared to the reference example.
4 Design Process

Design with respect to DIN EN 1992-1-1:2004 (NA) [2] [3]:

4.1 Exposure class
For concrete inside buildings with low air humidity → XC1
Min. concrete class → C 16/20
Chosen concrete class → C 37/30
\[ c_{\text{nom}} = c_{\text{min}} + c_{\text{dev}} \]
\[ c_{\text{nom}} = 10 + 10 = 20 \text{ mm} \]

4.2 Actions

Dead loading
\[ G_{k,1} = 400 \text{ kN} \]
\[ G_{k,2,1} = 0.40 \cdot 0.45 \cdot 6.20 \cdot 25 \text{ kN/m}^3 = 27.9 \text{ kN} \]
\[ G_{k,2,2} = 0.40 \cdot 0.17 \cdot 1.90 \cdot 25 \text{ kN/m}^3 = 3.2 \text{ kN} \]
\[ \sum G_{k,i} = 431 \text{ kN} \]

Variable loading
Snow: \[ Q_{k,s} = 68 \text{ kN} \]
Wind: \[ w_e = c_{pe} \cdot q(z_e) \]
\[ q_{k,w,1} = +0.7 \cdot 0.95 = +0.665 \text{ kN/m}^2 \]
\[ q_{k,w,2} = -0.3 \cdot 0.95 = -0.285 \text{ kN/m}^2 \]

Distance between columns is \[ \alpha = 6.5 \text{ m} \]

4.3 Materials

Concrete
Class C 30/37
\[ \gamma_c = 1.50 \]
\[ f_{ck} = 30 \text{ MPa} \]
\[ f_{cd} = a_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 30 / 1.5 = 17.0 \text{ MPa} \]

\textit{The tools used in the design process are based on steel stress-strain diagrams, as defined in [2] 3.2.7(2), Fig. 3.8.}

\textit{The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [2], [3], unless otherwise specified.}
Steel

Class B 500B

\( \gamma_s = 1.15 \)

\( f_{yk} = 500 \text{ MPa} \)

\( f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78 \text{ MPa} \)

4.4 Buckling length

Buckling length around the y-axis:

\[
l_{0,y} = \beta_y \cdot l_{col}
\]

\( l_{col} = 6.20 \text{ m} \)

\( \beta_y = 2.1 \leftarrow \text{approximated} \)

\( l_{0,y} = 6.2 \cdot 2.1 = 13.0 \text{ m} \)

\( \lambda_y = \frac{l_{y,0}}{l_y} = 13.0 \cdot \sqrt{12}/0.45 = 100 \)

Buckling length around the z-axis:

\[
l_{0,z} = \beta_z \cdot l_{col}
\]

\( l_{col} = 6.20 \text{ m} \)

\( \beta_z = 1.0 \leftarrow \text{pinned on both sides} \)

\( l_{0,z} = 6.2 \cdot 1.0 = 6.2 \text{ m} \)

\( \lambda_z = \frac{l_{z,0}}{l_z} = 6.2 \cdot \sqrt{12}/0.40 = 54 \)

Slenderness criterion for isolated member

\[
n = \frac{N_{Ed}}{A_c \cdot f_{cd}} = \frac{0.684}{0.40 \cdot 0.45 \cdot 17.0} = 0.224
\]

\( n \) is the relative normal force, (NDP) 5.8.3.1 (1)

\( l_{col}, \text{height of the column between the idealized fixing points.} \)

\( \beta_y, \text{ buckling factor} \)

\( \lambda_y, \text{ slenderness} \)

\( l_{col}, \text{height of the column between the idealized fixing points.} \)

\( \beta_z, \text{ buckling factor} \)

\( \lambda_z, \text{ slenderness} \)

Figure 3: Column properties
\[
\lambda_{\text{lim}} = \frac{16}{\sqrt{n}} = \frac{16}{0.224} = 34 \text{ for } n = 0.224 < 0.41
\]

Because \( \lambda_y = 100 \) and \( \lambda_z \) are \( > \lambda_{\text{lim}} = 34 \) second order effects should be taken into account in both directions (y and z).

### 4.5 Imperfections

For isolated members, the effect of imperfection may be taken into account:

\[
e_i = \theta \cdot \frac{l_0}{2}
\]

\[
\theta = \theta_0 \cdot \alpha_h \cdot \alpha_m
\]

\[
\theta_0 = \frac{1}{200}
\]

\[
\alpha_h = \frac{2}{\sqrt{l}} = \frac{2}{\sqrt{6.2}} = 0.803 < 1.0
\]

\[
\alpha_m = \sqrt{0.5 \cdot (1 + 1/m)} = \sqrt{0.5 \cdot (1 + 1/4)} = 0.79
\]

Bending about the y-axis:

\[
l_{0,y} = 13.00 \text{ m}
\]

\[
\theta_i = 0.803 \cdot 0.79 \cdot \frac{1}{200} = 0.00317 \approx 1/315
\]

\[
e_{a,z} = \frac{13000}{2 \cdot 315} = 21 \text{ mm}
\]

Bending about the z-axis:

\[
l_{0,z} = 6.20 \text{ m}
\]

\[
\theta_i = 0.803 \cdot 1.0 \cdot \frac{1}{200} = 0.00402 \approx 1/250
\]

\[
e_{a,y} = \frac{6200}{2 \cdot 250} = 13 \text{ mm}
\]

### 4.6 Min. and max. required reinforcement

\[
A_{s,\text{max}} = 0.09 \cdot 40 \cdot 45 = 162 \text{ cm}^2
\]

\[
A_{s,\text{min}} = 0.15 \cdot \frac{|N_{Ed}|}{f_{yd}} = 10^4 \cdot 0.15 \cdot 0.684/435 = 2.35 \text{ cm}^2
\]

Constructive: 6 \( \phi 12 \): \( A_{s,\text{min}} = 6.79 \text{ cm}^2 \)

### 4.7 Design of longitudinal reinforcement

**Nominal curvature method for y-y direction**

The design is approached by using the nominal curvature method. This method is primarily suitable for isolated members with constant normal force and a defined effective length \( l_0 \). The method gives a nominal
second order moment based on a deflection, which in turn is based on the effective length and an estimated maximum.

Design moment $M_{Ed}$:

$$M_{Ed} = M_{0Ed} + M_2 = N_{Ed} \cdot (e_0 + e_i + e_2)$$

$$K_1 = 1 \text{ for } \lambda > 35$$

$$n_u = 1 + \frac{f_{cd} \cdot A_c}{f_{yd} \cdot A_s}$$

In the reference example $A_s = 25.1 \, \text{cm}^2$ value is roughly estimated. The formula for $\omega$ value is NOT correct.

In SOFiSTiK we use $\omega = \frac{f_{yd} \cdot A_s}{f_{cd} \cdot A_c}$

$$n_u = 1 + \frac{17.0 \cdot 0.40 \cdot 0.45}{435 \cdot 25.1 \cdot 10^{-4}}$$

$$n_u = 1 + 3.06 \cdot 1.09 = 3.81$$

$$n_{bal} = 0.4$$

$$\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{435}{200000} = 2.175 \cdot 10^3$$

Statistical height:

$$d = h - c_{v,s} - \phi_{v,s} - \frac{\phi_i}{2}$$

$$d = 450 - 20 - 10 - \frac{16}{2} \approx 410 \, \text{mm}$$

Cross-section: $b/h/d = 400/450/410 \, \text{mm}$

$$\frac{d_1}{h} = \frac{(450 - 410)}{450} = 0.09 \approx 0.10$$

Table 3: Results

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
<th>Comb 1</th>
<th>Comb 2</th>
<th>Comb 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_y$</td>
<td>[kN]</td>
<td>74.40</td>
<td>90.40</td>
<td>100.00</td>
</tr>
<tr>
<td>$N$</td>
<td>[kN]</td>
<td>-684.00</td>
<td>-431.00</td>
<td>-633.00</td>
</tr>
<tr>
<td>$e_0$</td>
<td>[mm]</td>
<td>109</td>
<td>210</td>
<td>158</td>
</tr>
<tr>
<td>$e_1$</td>
<td>[mm]</td>
<td>21</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>$e_0 + e_i$ [mm]</td>
<td>[mm]</td>
<td>130</td>
<td>231</td>
<td>179</td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td>0.224</td>
<td>0.141</td>
<td>0.207</td>
</tr>
<tr>
<td>$K_r$</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 3: (continued)

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
<th>Comb 1</th>
<th>Comb 2</th>
<th>Comb 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e_2 = K_1 \cdot \frac{1}{r} \cdot \frac{l_0^2}{10})</td>
<td>[mm]</td>
<td>199</td>
<td>199</td>
<td>199</td>
</tr>
<tr>
<td>(e_{\text{tot}} = e_0 + e_i + e_2)</td>
<td>[mm]</td>
<td>329</td>
<td>430</td>
<td>378</td>
</tr>
<tr>
<td>(M_{\text{Ed}} = N_{\text{Ed}} \cdot e_{\text{tot}})</td>
<td>[kNm]</td>
<td>225</td>
<td>185</td>
<td>239</td>
</tr>
<tr>
<td>(\mu_{\text{Ed}} = M_{\text{Ed}}/(b \cdot d^2 \cdot f_{\text{cd}}))</td>
<td></td>
<td>0.16</td>
<td>0.13</td>
<td>0.174</td>
</tr>
<tr>
<td>(\omega_{\text{tot}})</td>
<td></td>
<td>0.19</td>
<td>0.13</td>
<td>0.174</td>
</tr>
<tr>
<td>(A_{s,\text{tot}} = \omega_{\text{tot}} \cdot b \cdot h \cdot \frac{f_{\text{cd}}}{f_{\text{yd}}})</td>
<td>[cm²]</td>
<td>13.40</td>
<td>12.70</td>
<td>16.20</td>
</tr>
</tbody>
</table>

Creep and shrinkage

\(\phi_{\text{ef}} = \phi(\infty, t_0) \cdot \frac{M_{\text{0Ed}}}{M_{\text{0Ed}}^p}\)

\(K_{\phi} = 1 + \beta \cdot \phi_{\text{ef}} \geq 1.0\)

\(\beta = 0.35 + \frac{f_{\text{ck}}}{200} - \frac{\lambda}{150} \geq 0\)

\(\beta = 0.35 + \frac{300}{200} - \frac{100}{150} = -0.17 < 0\)

\(K_{\phi} = 1 \rightarrow\) Creep and shrinkage is neglected!

Nominal curvature method for z-z direction

The column will be designed as a non-reinforcement column for z-z direction:

Buckling length:

\(l_{0,z} = 6.20\ m\)

Slenderness:

\(\lambda_z = 54 < \lambda_{\text{max}} = 86\)

Limits for the second order theory:

\(\frac{l_{\text{col}}}{h} = \frac{6.20}{0.40} = 15.50 > 2.5\)

The column should be investigated for the second order effects.

Design resistance normal force \(N_{\text{Rd}}\):

\(N_{\text{Rd}} = b \cdot h_w \cdot f_{\text{cd,pl}} \cdot \Phi\)

\(\Phi = 1.14 \cdot (1 - 2 \cdot \frac{e_{\text{tot}}}{h_w}) - 0.02 \cdot \frac{l_0}{h_w} \leq 1 - \frac{2 \cdot e_{\text{tot}}}{h}\)
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$e_{\text{tot}} = e_0 + e_1 = 0 + 13 = 13 \text{ mm}$

$\Phi = 1.14 \cdot (1 - 2 \cdot \frac{13}{400}) - 0.02 \cdot \frac{6200}{400} \leq 1 - 2 \cdot \frac{13}{400}$

$\Phi = 0.756 \leq 0.94$

$f_{cd,p,l} = \frac{0.70 \cdot 30}{1.5} = 14.0 \text{ N/mm}^2$

$N_{Rd} = 0.45 \cdot 0.40 \cdot 14.0 \cdot 0.756 = 1.9 \text{ sMN} > |N_{\text{Ed}}| = 0.684 \text{ MN}$

### 4.8 Design of shear reinforcement

Shear design on bottom of column:

$V_{\text{Ed}} = -32.0 \text{ kN}$

$N_{\text{Ed}} = 431 \text{ kN}$

$d = 0.41 \text{ m}$

$V_{\text{Ed,ref}} = 32.0 - 0.41 \cdot 6.48 = 29.3 \text{ kN}$

The design value of the shear resistance $V_{Rd,c}$

$V_{Rd,c} = \left[ \frac{0.15}{\gamma_c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_{cp} \right] \cdot b_w \cdot d$

$V_{Rd,c,\text{min}}$

$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$

$k = 1 + \sqrt{\frac{200}{410}} = 1.70$

$\rho_l = \frac{A_{sl}}{b_w \cdot d} = \frac{8.04}{40 \cdot 41} = 0.0049 < 0.02$

$f_{ck} = 30 \text{ N/mm}^2$

$\sigma_{cp} = \frac{N_{\text{Ed}}}{A_c} = \frac{0.431}{0.40 - 0.45} = 2.39 \text{ MN/m}^2$

$C30/37 \rightarrow 0.2 \cdot f_{cd} = 3.4 \text{ MPa}$

$V_{Rd,c} = \left[ \frac{0.15}{1.5} \cdot 1.70 \cdot (100 \cdot 0.49 \cdot 30)^{1/3} + 0.12 \cdot 2.39 \right] \cdot 0.40 \cdot 0.41 \geq 0.115 \text{ MN}$

$V_{\text{Ed,red}} = 29.3 \text{ kN} < V_{Rd,c} = 115 \text{ kN}$

$\rightarrow$ no shear reinforcement required!
5 Conclusion

This example shows the calculation of column design by using the nominal curvature method. Various ways of reference calculations are demonstrated, in order to compare the reference results to SOFiSTiK.

The main reason for minor deviations (See Tab. 3 in Section 3) is that in the reference benchmark, the 2nd order effects and the reinforcement are approximated by using analytical formulas. In SOFiSTiK, the $K_r$ value is iterated until $K_r < 1 \cdot 10^{-4}$, if this condition is met, then the iteration is stopped. With a hand calculation this is very difficult to achieve.

![Figure 4: Iteration](image)

The general approach of the nominal curvature is listed as following steps:

1. Calculation of the imperfection $e_i$, slenderness $\lambda$ and inner lever arm $d_{y,z}$

2. Find the critical deflection $e_0$, which depends on the current loadcase combination. Consider different end-moments according EN 1992-1-1:2004, 5.8.8.2 (2), Eq. 5.32

3. Start iteration of design moment $M_{Ed}$ (Theory II. Order). The iteration will stop, when the coefficient $K_r$ achieved convergence.
   
   (a) Validation, if uni-/biaxial design can be applied
   
   (b) Calculate the coefficient $K_r$ and the eccentricity $e_2$
   
   (c) Calculate actual design moment $M_{Ed} = N_{Ed} \cdot (e_0 + e_i + e_2)$
   
   (d) Calculate required reinforcement with the program AQB

4. Additional cross section design on different locations (without imperfection and theory II.O.)

5. Result: decisive design moment, required reinforcement and utilization factor.
6 Literature

