Benchmark Example No. 5

Design of a Rectangular CS for Double Bending and Axial Force

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The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.
1 Problem Description

The problem consists of a rectangular section, as shown in Fig. 1. The cross-section is designed for double axially bending moments $M_{E_d y}$, $M_{E_d z}$ and a compressive force $N_{E_d}$.

![Figure 1: Problem Description](image)

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to double bending with axial force. The content of the problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Basic assumptions for section design (Section 6.1)
- Reinforcement (Section 9.3.1.1, 9.2.1.1)

![Figure 2: Stress and Strain Distributions in the Design of Doubly Reinforced Cross-sections](image)

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).
3 Model and Results

The rectangular cross-section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], under double axial bending and an axial compressive force of 1600 \( kN \). The calculation steps with a commonly used design method [3] [4] are presented below and the results are given in Table 2. Here, it has to be mentioned that the standard methods employed in order to calculate the reinforcement are approximate, and therefore deviations often occur.

Table 1: Model Properties

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 35/45</td>
<td>( h = 50.0 , cm )</td>
<td>( M_{Edy} = 500 , kNm )</td>
</tr>
<tr>
<td>B 500A</td>
<td>( b_1 = b_2 = 5.0 , cm ) ( d_1 = d_2 = 5.0 , cm ) ( b = 40 , cm )</td>
<td>( M_{Edz} = 450 , kNm ) ( N_{Ed} = -1600 , kN )</td>
</tr>
</tbody>
</table>

Table 2: Results

<table>
<thead>
<tr>
<th>( A_{s,tot} ,[cm^2/m] )</th>
<th>SOF Interaction Diagram [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>115.9</td>
<td>113.1</td>
</tr>
</tbody>
</table>
4 Design Process

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

$f_{ck} = 30 \text{ MPa}$

$f_{cd} = \alpha_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 35 / 1.5 = 19.8 \text{ MPa}$

$f_{yk} = 500 \text{ MPa}$

$f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78 \text{ MPa}$

Design Load:

$N_{Ed} = -1600 \text{ kN}$

$M_{Edy} = 500 \text{ kNm}$

$M_{Edz} = 450 \text{ kNm}$

Design with respect to Interaction diagram for Double Bending with axial force for rectangular cross-sections:

$\mu_{Edy} = \frac{M_{Ed}}{b \cdot h^2 \cdot f_{cd}} = \frac{500 \cdot 10^{-3}}{0.40 \cdot 0.50^2 \cdot 19.8} = 0.252$

$\mu_{Edz} = \frac{M_{Ed}}{b \cdot h^2 \cdot f_{cd}} = \frac{450 \cdot 10^{-3}}{0.40 \cdot 0.50^2 \cdot 19.8} = 0.284$

$\nu_{Ed} = \frac{N_{Ed}}{b \cdot h^2 \cdot f_{cd}} = \frac{-1600 \cdot 10^{-3}}{0.40 \cdot 0.50^2 \cdot 19.8} = -0.403$

From design chart $\rightarrow \omega_{tot} = 1.24$ for:

- $d_1/h = d_2/h = 0.05/0.5 = 0.10$
- $b_1/b = b_2/b = 0.05/0.4 = 0.08 \approx 0.10$
- $\nu = -0.4$
- $\mu_1 = \max[\mu_{Edy}; \mu_{Edz}] = 0.284$
- $\mu_2 = \min[\mu_{Edy}; \mu_{Edz}] = 0.252$

$A_{s,tot} = \omega_{tot} \cdot \frac{b \cdot h}{f_{yd} / f_{cd}} = 113.1 \text{ cm}^2$

$A_{s,tot}/4 = 28.28 \text{ cm}^2$

1The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

2The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.
5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular beam cross-section under double axial bending with compressive axial force. It has been shown that the results are reproduced with excellent accuracy.

6 Literature


