Benchmark Example No. 6

Design of a Rectangular CS for Shear Force

SOFiSTiK | 2020
1 Problem Description

The problem consists of a rectangular section, symmetrically reinforced for bending, as shown in Fig. 1. The cross-section is designed for shear force $V_{Ed}$ and the required shear reinforcement is determined.

![Figure 1: Problem Description](image1)

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to shear force. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1] [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear design (Section 6.2)
- Reinforcement (Section 9.2.2)

![Figure 2: Shear Reinforced Members](image2)

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).
3 Model and Results

The rectangular section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], under shear force of 343.25 kN. The reference calculation steps are presented below and the results are given in Table 2. Then, the same section is designed with respect to EN 1992-1-1:2004 [3]. The same angle $\theta (= 1.60)$ is chosen, as calculated with respect to DIN EN 1992-1-1:2004, in order to compare the results. If no $\theta$ value is input, then the calculation starts with the upper limit $\cot \theta = 2.5$ and through an optimization process the right angle is selected. In this case, the reinforcement is determined with $\cot \theta = 2.5$, giving a shear reinforcement of $7.80 \ cm^2/m$. Also in order to demonstrate that the correct value of $V_{Rd,max} = 734.4 \ kN$ (reference value) with respect to DIN EN 1992-1-1:2004 is calculated in SOFiSTiK, we input a design shear force of 734.3 delivering a shear reinforcement, but when a value of 734.4 is input then AQB gives the warning of 'no shear design possible' showing that the maximum shear resistance is exceeded.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Geometric Properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 30/37</td>
<td>$h = 50.0 \ cm$</td>
<td>$V_{Ed} = 343.25 \ kN$</td>
</tr>
<tr>
<td>B 500A</td>
<td>$b = 30 \ cm, d = 45.0 \ cm$</td>
<td>$A_{s,tot} = 38.67 \ cm^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$A_{s,tot}[cm^2/m]$</th>
<th>Design Code</th>
<th>SOF.</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DIN EN [1]</td>
<td>12.84</td>
<td>12.84</td>
</tr>
<tr>
<td></td>
<td>EN [3]</td>
<td>12.18</td>
<td>12.18</td>
</tr>
</tbody>
</table>
4 Design Process

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

$f_{ck} = 30 \text{ MPa}$

$f_{cd} = \alpha_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 30 / 1.5 = 17.0 \text{ MPa}$

$f_{yk} = 500 \text{ MPa}$

$f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78 \text{ MPa}$

Design Load: $V_{Ed} = 343.25 \text{ kN}$

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2].

Design with respect to EN 1992-1-1:2004 [3].

$z = \max \{d - c_{V,i} - 30 \text{ mm}; \ d - 2 \ c_{V,i}\}$

$z = \max \{384; 378\} = 384 \text{ mm}$

$1.0 \leq \cot \theta \leq \frac{1.2 + 1.4 \ \sigma_{cd} / f_{cd}}{1 - V_{Rd,cc} / V_{Ed}} \leq 3.0$

$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot \left(1 - 1.2 \frac{\sigma_{cd}}{f_{cd}}\right) \cdot b_w \cdot z$

$V_{Rd,cc} = 0.5 \cdot 0.48 \cdot 30^{1/3} \cdot (1 - 0) \cdot 0.3 \cdot 0.384$

$V_{Rd,cc} = 0.08591 \text{ MN} = 85.91 \text{ kN}$

$\cot \theta = \frac{1.2 + 0}{1 - 85.91 / 343.25} = 1.60$

$A_{sw,requ} / s = V_{Ed} / (f_{ywd} \cdot z \cdot \cot \theta) = 12.84 \text{ cm}^2 / \text{m}$

$V_{Rd,max} = b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot \theta + \tan \theta)$

$V_{Rd,max} = 0.3 \cdot 0.384 \cdot 0.75 \cdot 17 / (1 + 1) = 734.4 \text{ kN}$

Design with respect to EN 1992-1-1:2004 [3].

$z = 0.9 \cdot d$

$z = 0.9 \cdot 450 = 405 \text{ mm}$

$1.0 \leq \cot \theta \leq 2.5 \rightarrow \cot \theta = 1.60$ (choose for comparison)

$A_{sw,requ} / s = V_{Ed} / (f_{ywd} \cdot z \cdot \cot \theta) = 12.18 \text{ cm}^2 / \text{m}$

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1 The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7(2), Fig. 3.8, which can be seen in Fig. 3.

2 The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

3 The sections mentioned in the margins refer to EN 1992-1-1:2004 [3], unless otherwise specified.

(NDP) Eq. 6.7DE: The angle $\theta$ should be limited by Eq. 6.7DE

$6.2.3(3)$: Eq. 6.6

$V_{Rd} = f_{ydk} / \gamma_s = 435 \text{ MPa}$

(NDP) Eq. 6.2.3 (3): Eq. 6.9

Maximum shear force $V_{Rd,max}$ occurs for $\theta = 45^\circ$: $\cot \theta = \tan \theta = 1$

$v_1 = 0.75 \cdot v_2 = 0.75$, $v_2 = 1$ for $\leq 50^\circ$

(NDP) Eq. 6.7N

The angle $\theta$ should be limited by Eq. 6.7N

$6.2.3(1)$: Inner lever arm $z$

$6.2.3(2)$: Eq. 6.7N
5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular cross-section under shear force. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

