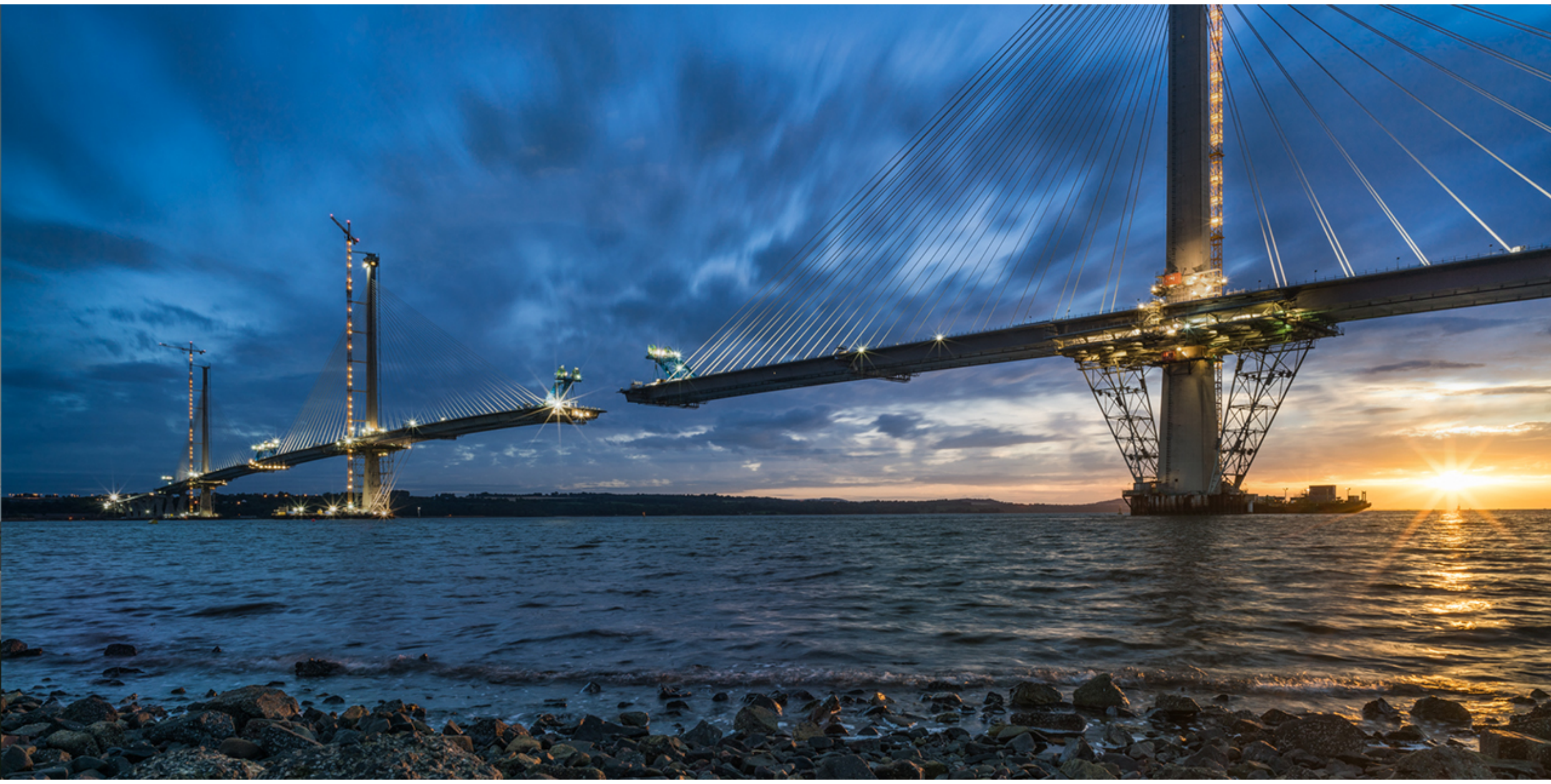

Benchmark Example No. 45

One-Dimensional Soil Consolidation

SOFiSTiK | 2020

VERiFiCATiON
BE45 One-Dimensional Soil Consolidation

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SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Project: Queensferry Crossing | Photo: Bastian Kratzke

Overview	
Element Type(s):	C2D
Analysis Type(s):	STAT
Procedure(s):	
Topic(s):	SOIL
Module(s):	TALPA
Input file(s):	soil_1d_consolidation.dat

1 Problem Description

In the following example a one-dimensional consolidation problem has been analyzed. The soil layer is subjected to an uniform loading of the intensity p_0 acting on the surface. Base of the soil is rigidly fixed while the sides are laterally constrained. Only the soil surface is allowed to drain. Geometry, load and boundary conditions are depicted in Fig. 1. The soil material is elastic, isotropic and saturated with water. The surface settlements and pore excess pressures for the two extreme cases (time zero and time infinity) of the consolidation process are compared to the analytical solution.

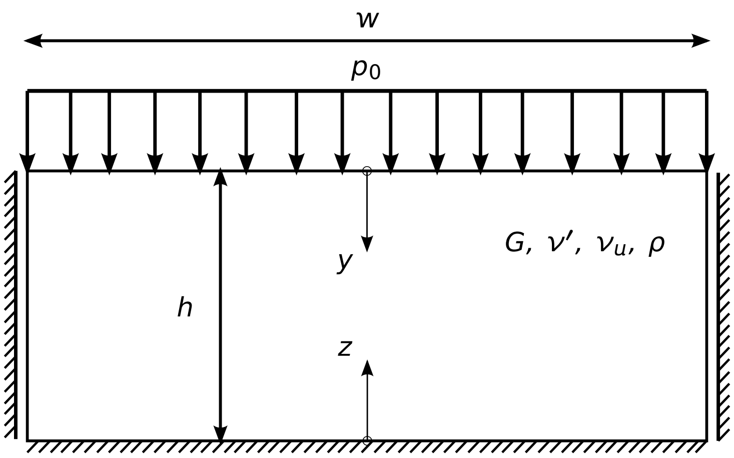

Figure 1: Problem Definition

2 Reference Solution

The analytical solution to the problem was given by Terzaghi in 1925 [1]. The solution assumes that the soil is saturated with water, the soil and water are non-deformable, the volume change takes place only on the account of the water drainage and the Darcy's filtration law applies. Then the differential equation of the one-dimensional process of consolidation for the excess water pressure p_{we} can be written as [2]:

$$\frac{\partial p_{we}}{\partial t} = c_v \frac{\partial^2 p_{we}}{\partial z^2}, \tag{1}$$

where:

$c_v = k \cdot E_s / \gamma_w$	coefficient of consolidation,
E_s	stiffness modulus,
k	coefficient of permeability,

γ_w	unit weight of water,
h	soil thickness,
$z = h - y$	elevation.

Taking into account the initial and boundary conditions for the problem illustrated by Fig. 1

$$t = 0 \quad \text{and} \quad 0 \leq z < h \quad \Rightarrow \quad p_{we} = p_0, \tag{2a}$$

$$0 \leq t \leq \infty \quad \text{and} \quad z = 0 \quad \Rightarrow \quad \frac{\partial p_{we}}{\partial z} = 0, \tag{2b}$$

$$0 \leq t \leq \infty \quad \text{and} \quad z = h \quad \Rightarrow \quad p_{we} = 0, \tag{2c}$$

$$t = \infty \quad \text{and} \quad 0 \leq z \leq h \quad \Rightarrow \quad p_{we} = 0, \tag{2d}$$

the Eq. 1 can be analytically solved for p_{we} as a function of the time t and the elevation $z = h - y$

$$\frac{p_{we}(t,z)}{p_0} = \frac{4}{\pi} \cdot \sum_{j=0}^{\infty} \frac{1}{2j+1} \cdot \sin\left((2j+1)\frac{\pi}{2}\frac{z}{h}\right) \cdot e^{-(2j+1)^2\pi^2/4 \cdot T_v} \tag{3}$$

where:

p_0	surface pressure,
$T_v = c_v/h^2 \cdot t$	time factor.

With the known change of excess pore pressure with respect to time, the settlement due to consolidation $s(t)$ at time t can be determined

$$s(t) = \frac{p_0 h}{E_s} \cdot \left[1 - \frac{8}{\pi^2} \cdot \sum_{j=0}^{\infty} \frac{1}{(2j+1)^2} \cdot e^{-(2j+1)^2\pi^2/4 \cdot T_v}\right]. \tag{4}$$

For the time infinity, the excess pore pressures will completely dissipate (see Eq. 2d) and the final settlements due to consolidation s_∞ will be

$$s_\infty = s(t = \infty) = \frac{p_0 h}{E_s}. \tag{5}$$

3 Model and Results

Elastic, isotropic soil under undrained and drained conditions has been analyzed. Material, geometry and loading properties are summarized in Table 1. The undrained soil condition is considered with the help of the method based on the undrained effective stress (σ') analysis using effective material parameters. G and ν' are effective soil parameters, while B represents the Skempton's B-parameter. Self-weight is not taken into consideration.

Table 1: Model Properties

Material	Geometry	Loading
G, $\nu' = 0.0$	h	p_0
$B = 0.998$	$w = h$	

Table 1: (continued)

Material	Geometry	Loading
$\rho = 0.0\ kg/m^3$		

Finite element mesh of the model is shown in Fig. 2. Mesh is regular and consist of quadrilateral finite elements.

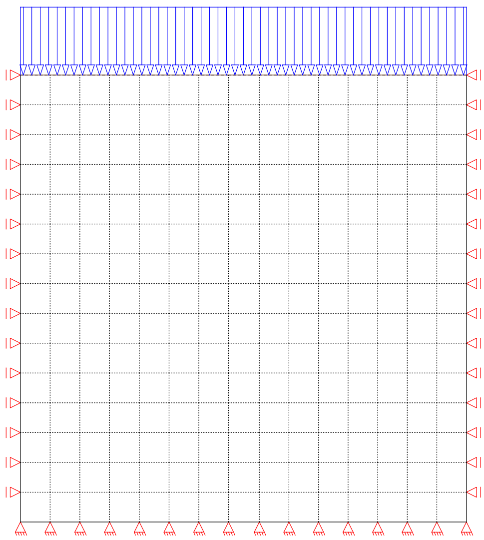

Figure 2: Finite Element Model

The results are summarized in the Table 2. Final settlement of the surface of the soil due to consolidation s_∞ is compared to the analytical solution given by Eq. 5. The excess water pressures p_{we} for the time zero ($T_v = 0$, undrained) and time infinity ($T_v = \infty$, drained) are compared to the analytical solutions from Eqs. 2a and 2d.

Table 2: Results

		$T_v = 0$		$\|e\|$	$T_v = \infty$		$\|e\|$
		SOF.	Ref.	$[-]$	SOF.	Ref.	$[-]$
$s(T_v) \cdot E_s/(p_0 h)$	$[-]$		–		1.0	1.0	0.0
$p_{we}(T_v)/p_0$	$[-]$	0.994	1.000	0.006	0.000	0.000	0.000

4 Conclusion

The example verifies that the settlements and excess pore pressures for initial ($t = 0$) and finial ($t = \infty$) time of the consolidation process obtained by the finite element method are in a good agreement with the analytical solution.

5 Literature

[1] K. Terzaghi. *Erdbaumechanik auf bodenphysikalischer Grundlage*. Leipzig (usw.): F. Deuticke, 1925.

[2] K. Terzaghi. *Theoretical Soil Mechanics*. Wiley, 1948.