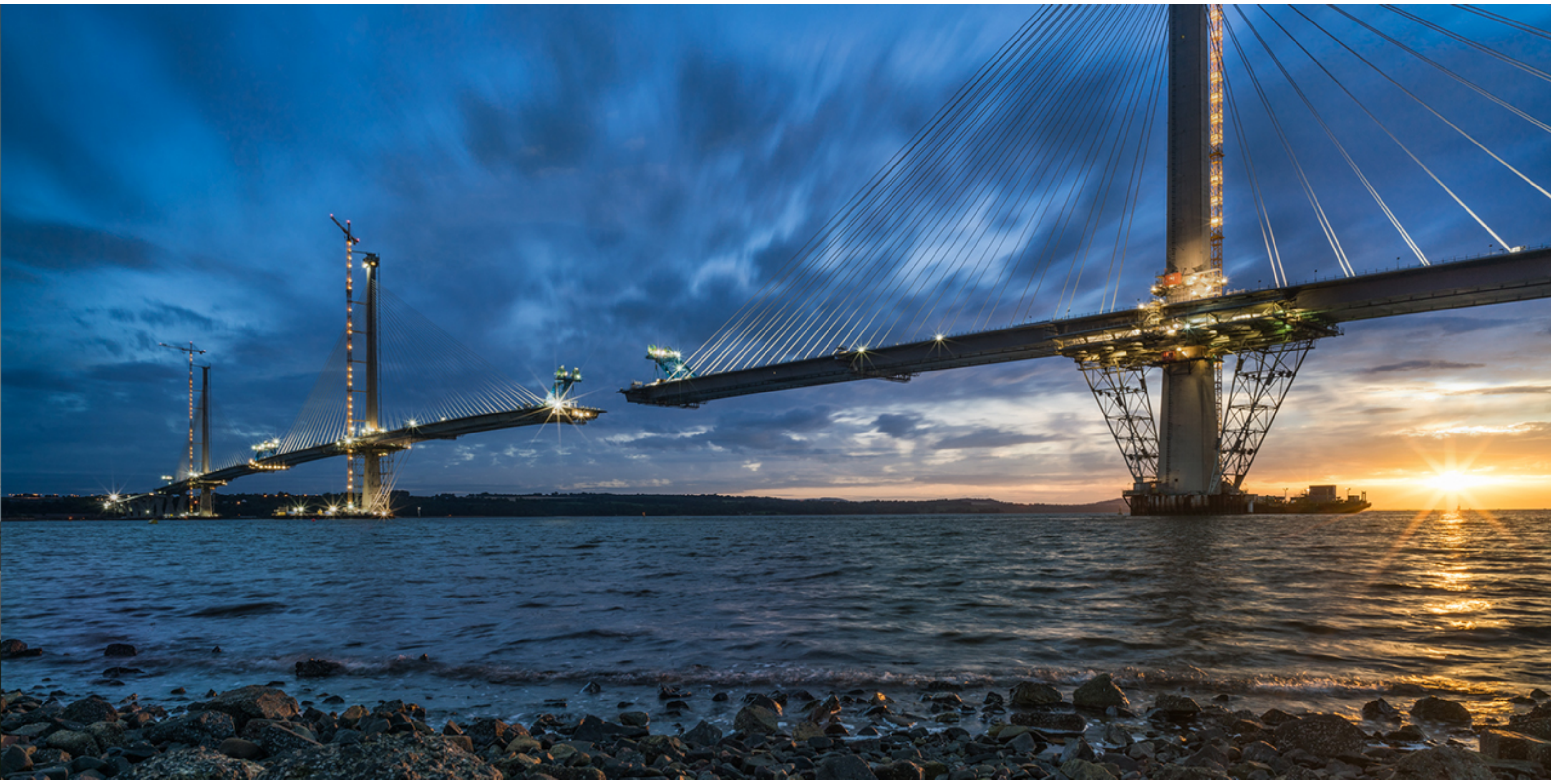

Benchmark Example No. 50

A Circular Cavity Embedded in a Full-Plane Under Impulse Pressure

SOFiSTiK | 2020

VERiFiCATiON
BE50 A Circular Cavity Embedded in a Full-Plane Under Impulse Pressure

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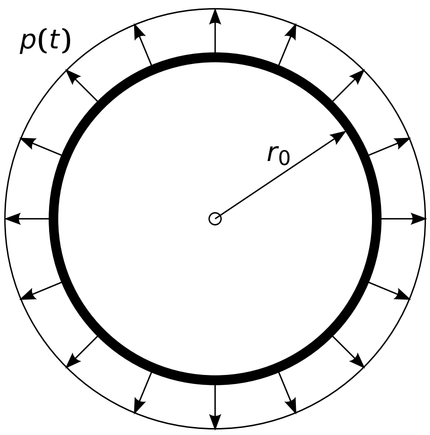
Front Cover

Project: Queensferry Crossing | Photo: Bastian Kratzke

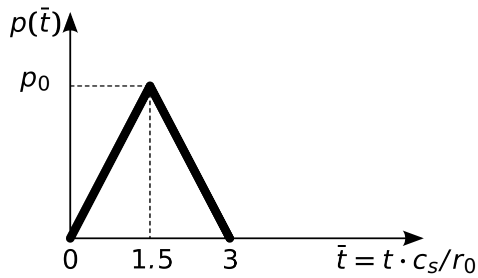
Overview	
Element Type(s):	C2D
Analysis Type(s):	DYN
Procedure(s):	
Topic(s):	SOIL
Module(s):	DYNA
Input file(s):	sbfem_2d_cric_cavity.dat

1 Problem Description

This example addresses a circular cavity with radius r_0 embedded in a full-plane subjected to a radial pressure $p(t)$ (Fig. 1). The full-plane is assumed to be elastic, homogeneous, isotropic, without material damping which stretches to infinity and it is modeled with the help of the Scaled Boundary Finite Elements (SBFE). Plane-strain condition is considered. Load is in a form of a triangular impulse and applied on the cavity wall (Fig. 1b). Radial displacement response of the cavity wall has been computed and compared to the analytical solution.



(a) Circular cavity embedded in a full-plane



(b) Pressure load

Figure 1: Problem Definition

2 Reference Solution

This problem is essentially a one dimensional problem which has an analytical solution [1]. The force-displacement relationship for this problem in frequency domain is given by

$$P(\omega) = S^{\infty}(\omega) \cdot u_r(\omega) \ , \tag{1}$$

where $\omega = 2\pi f$ represents the circular frequency, $P(\omega)$ is the *total force* applied on the cavity wall, $u_r(\omega)$ is the radial displacement and $S^{\infty}(\omega)$ is the dynamic-stiffness coefficient.

The dynamic-stiffness coefficient for this particular problem has an analytical expression and it reads

$$S^{\infty}(a_0) = \frac{2\pi G_0}{1-2\nu} \cdot \left[2(1-\nu)\frac{\lambda - 1 - F}{\lambda} - 2\nu + 2(1-\nu)\frac{H^{(2)}_{F+1}(\lambda a_0)}{H^{(2)}_{F}(\lambda a_0)} a_0 \right] \ , \tag{2}$$

where

G_0	shear modulus,
ν	Poisson's ratio,
ρ	mass density,
$c_s = \sqrt{G/\rho}$	shear wave velocity,
$c_p = c_s\sqrt{(2-2\nu)/(1-2nu)}$	P-wave velocity,
$a_0 = \omega r_0/c_p$	dimensionless frequency,
$\lambda = 2/(2-\alpha)$	coefficient,
α	non-homogeneity parameter of elasticity ($\alpha = 0$ for the homogeneous case),
$H_k^{(2)}$	the second kind Hankel function of the order k,
$F = \sqrt{(\lambda-1)^2 - \lambda^2 \dfrac{\nu(\alpha+1)-1}{1-\nu}}$	order of the Hankel function.

The static-stiffness coefficient K^{∞} is used to non-dimensionlize displacement response

$$K^{\infty} = \frac{2\pi G_0}{1-2\nu} \cdot \left[\alpha(1-\nu) - 2\nu + \sqrt{(\alpha(1-\nu)-2\nu)^2 + 4 - 8\nu}\right] . \tag{3}$$

The radial displacement response in frequency domain $u_r(\omega)$ is obtained by first making the Fourier transformation of the total triangular impulse load $P(\omega)$ (Fig. 1b) and then dividing it with the dynamic-stiffness coefficient $S^{\infty}(\omega)$ (Eq. 1). Finally the displacement response is transformed in the time domain ($u_r(t)$) using the inverse Fourier transformation.

3 Model and Results

Material, geometry and loading properties of the model are summarized in the Table 1. The plane-strain model of the full-pane is assumed to be elastic, homogeneous ($\alpha = 0$) and isotropic.

Table 1: Model Properties

Material	Geometry	Loading	Integration parameters
c_s, ρ, $\nu = 0.3$	r_0	$P(t) = 2\pi r_0 p(t)$	$\Delta t = 0.04 \cdot r_0/c_p$
$G_0 = \rho c_s^2$		$P_0 = 2\pi r_0 p_0$	$M, N, \theta = 1.4$

Load and the finite element model of the structure are depicted in Fig. 2. The structure is comprised solely of the 2-node line scaled boundary finite elements and the load is applied directly to the nodes of the boundary.

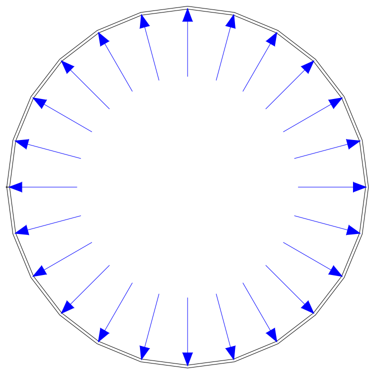


Figure 2: Finite Element Model

The transient radial displacement response of the cavity wall $u_r(t)$ has been computed using the Scaled Boundary Finite Element Method (SBFEM) in the time domain. The integration of the governing equations of the SBFEM is performed using the original discretization scheme (*const*) [1][2] and the extrapolation scheme from [3] based on the parameters M, N and θ [1].

The results in dimensionless form are plotted in Fig. 3 together with the analytical solution. The numerical results show excellent agreement with the analytical solution for all three cases.

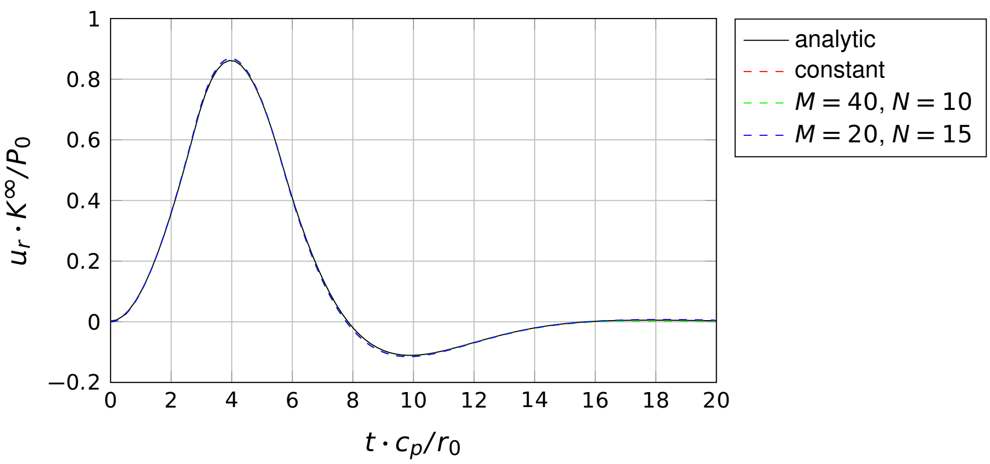


Figure 3: Radial displacement response

4 Conclusion

The example verifies the accuracy of the SBFEM method in modeling unbounded domain problems. The integration scheme for the solution of the governing equations of the SBFEM in time domain based on the work from [3] provides the solution with high computational efficiency and little loss of accuracy compared to the original method from [2].

5 Literature

[1] M.H. Bazyar. “Dynamic Soil-Structure Interaction Analysis Using the Scaled Boundary Finite-Element Method”. PhD thesis. Sydney, Australia: The University of New South Wales, School of Civil and Environmental Engineering, 2007.

[1] For the full description of the scheme based on the extrapolation parameter θ and the meaning of the integration parameters M, N and θ consult [3].

[2] J.P. Wolf and C. Song. *Finite-Element Modelling of Unbounded Media*. Chichester, UK: John Wiley and Sons, 1996.
[3] B. Radmanović and C. Katz. “A High Performance Scaled Boundary Finite Element Method”. In: *IOP Conference Series: Materials Science and Engineering* 10 (2010).