



Benchmark Example No. 32

Crack width calculation of reinforced slab acc. DIN EN 1992-1-1

VERiFiCATION
DCE-EN32 Crack width calculation of reinforced slab acc. DIN EN 1992-1-1

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The manual and the program have been thoroughly checked for errors. However, SOFiSTiK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.

Front Cover

Project: Queensferry Crossing | Photo: Bastian Kratzke

Overview

Design Code Family(s): DIN

Design Code(s): DIN EN 1992-1-1

Module(s): BEMESS

Input file(s): [crack_width_slab_din-en-1992-1-1.dat](#)

1 Problem Description

The problem consists of a flat slab, reinforced, as shown in Fig. 1. Loading always consisting of a bending moment M_{Ed} , the normal force is $N = 0$. The crack width is determined.

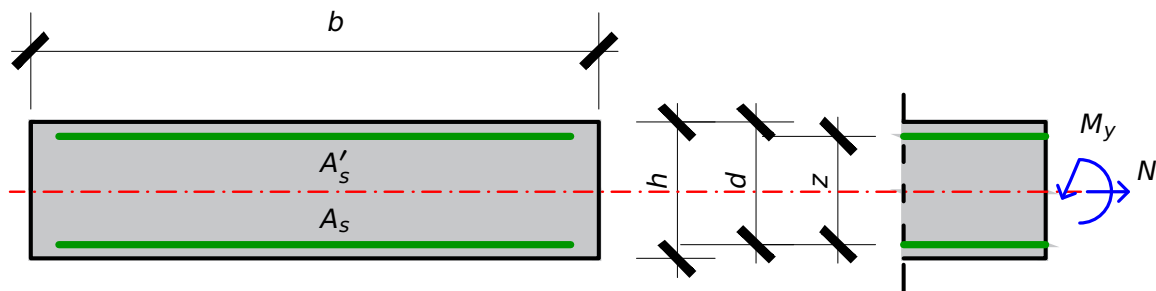


Figure 1: Geometry of slab, $b = 1 \text{ m}'$

2 Reference Solution

This example is concerned with the control of crack widths. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Basic assumptions for calculation of crack widths (Section 7.3.2, 7.3.3, 7.3.4)

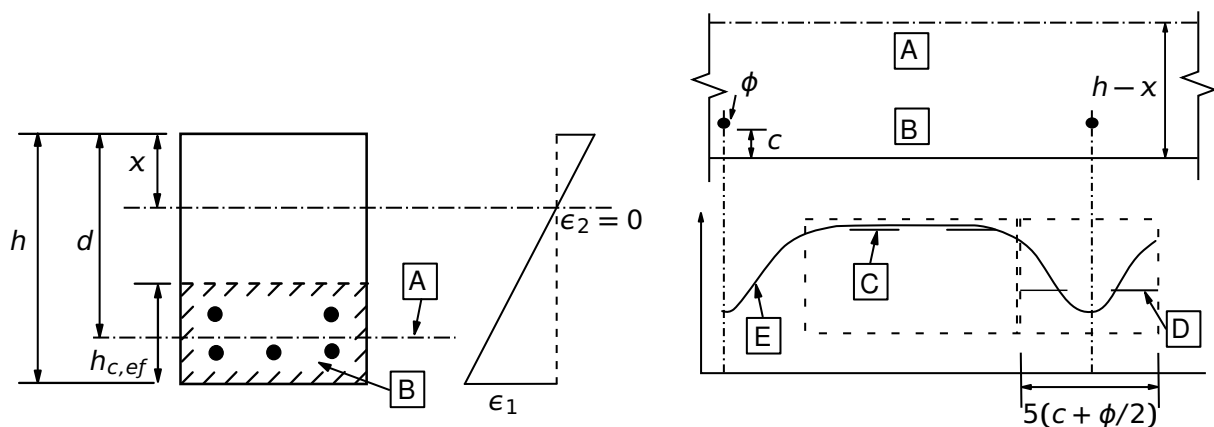


Figure 2: Stress and Strain Distributions in the Design of Cross-sections

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).

3 Model and Results

The slab, with properties as defined in Table 4, is to be designed for crack width, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1] [2]. The calculation steps with different loading conditions and calculated with different sections of DIN EN 1992-1-1:2004 + NA are presented below and the results are given in Table 3.

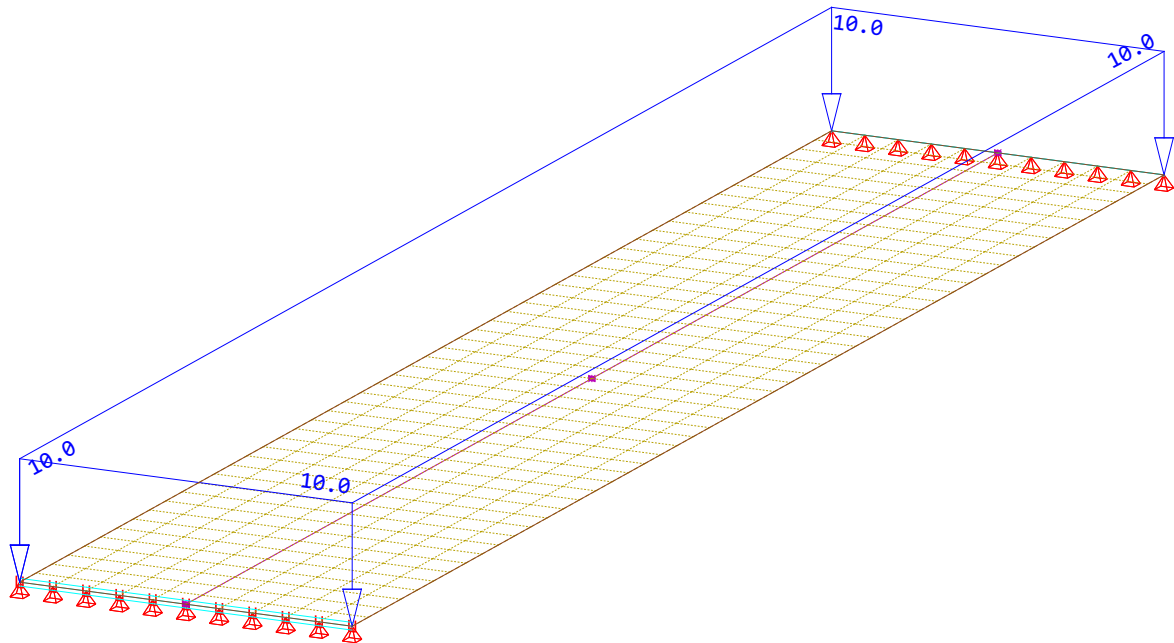


Figure 3: Model and Loading

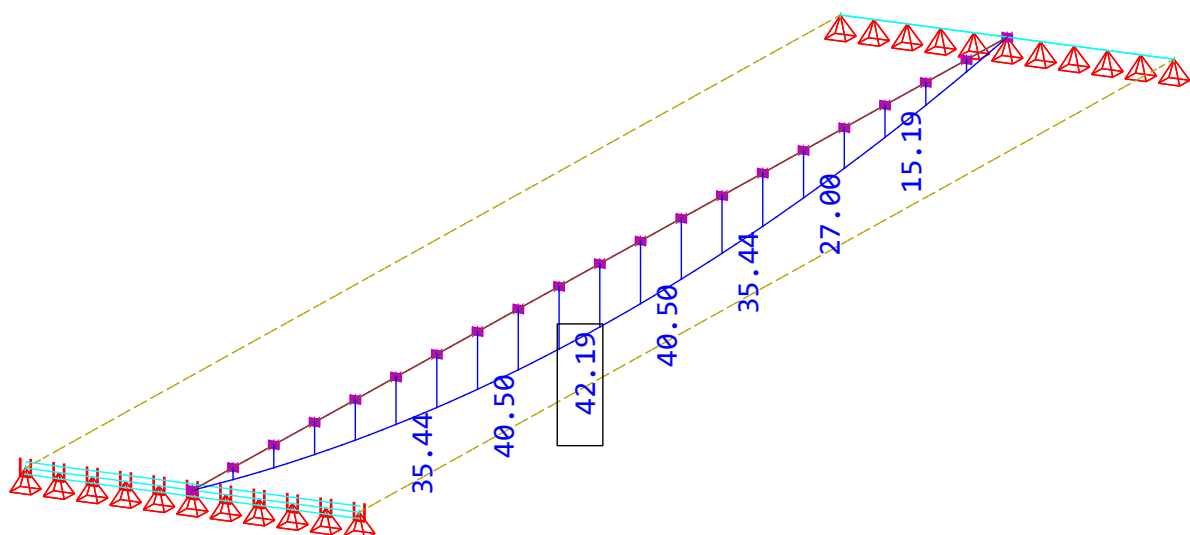


Figure 4: Model and internal forces - design element - $\gamma_G = 1.35$

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
C 25/30	$h = 24.0 \text{ cm}$	$M_{Ed} = 42.11 \text{ kNm}$
B 500A	$d = 21 \text{ cm}$	
	$b = 1000.0 \text{ cm}$	
	$\phi_s = 10.0 \text{ mm}, a_s = 5.50 \text{ cm}^2/\text{m}'$	
	$\phi_{s'} = 10.0 \text{ mm}$	
	$w_{k,req} = 0.3 \text{ mm}$	

Table 2: Results - Crack width calculation WITHOUT direct method (quad element 10226)

Case	SOF.	REF.
$\sigma_{s,tab} [\text{MPa}]$	303.87	303.87
$\phi_s [\text{mm}]$	10	10
$\phi_s^* [\text{mm}]$	11.337	11.337
$\sigma_s [\text{MPa}]$	303.87	303.87
$a_s [\text{cm}^2/\text{m}]$	7.07	7.34

Table 3: Results - Crack width calculation with direct method (quad element 10226)

Case	SOF.	REF.
$a_s = 5.5 \text{ cm}^2/\text{m}'$	$w_k = 0.434 \text{ mm}$	$w_k = 0.434 \text{ mm}$
$w_{k,req} = 0.3 \text{ mm}$	$a_s = 6.59 \text{ cm}^2/\text{m}$	$a_s = 6.6 \text{ cm}^2/\text{m}'$

4 Design Process ¹

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:²

4.1 Material

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

Tab. 3.1: Strength for concrete

$f_{ck} = 25 \text{ MPa}$

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{cd} = \alpha_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 25 / 1.5 = 14.17 \text{ MPa}$

3.2.2: (3)P: yield strength $f_{yk} = 500 \text{ MPa}$

$f_{yk} = 500 \text{ MPa}$

3.2.7: (2), Fig. 3.8

$f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 434.78 \text{ MPa}$

Design Load:

$M_{Ed} = 42.11 \text{ kNm}$

$N_{Ed} = 0.0$

4.2 Without direct calculation

7.3.3: Control of cracking without direct calculation

Examples calculated in this section are w.r.t. Table 7.2DE, here Table 7.3N is not relevant

(NDP) 7.3.2 (2): $f_{ct,eff}$ mean value of concrete tensile strength

Tab. 3.1: $f_{ctm} = 2.6 \text{ MPa}$ for C 25

(NDP) 7.3.3: Eq. 7.7.1DE: The maximum bar diameters should be modified for load action

$f_{ct,eff} = f_{ctm}$

$f_{ct,eff} = 2.56 \text{ MPa}$

$$\sigma_s = \frac{m_{Ed}}{A_s \cdot z}$$

$$\sigma_s = \frac{42.11 \cdot 10^{-3}}{5.5 \cdot 10^{-4} \cdot 0.9 \cdot 0.21} = 405.09 \text{ MPa}$$

$$\phi_s = \phi_s^* \cdot \frac{\sigma_s \cdot A_s}{4(h-d) \cdot b \cdot 2.9} \geq \phi_s^* \cdot \frac{f_{ct,eff}}{2.9}$$

$$\phi_s = 10 \text{ mm} = \phi_s^* \cdot \frac{405.09 \cdot 5.5}{4(24-21) \cdot 100 \cdot 2.9} = \phi_s^* \cdot 0.475$$

$$\phi_s = 10 = \phi_s^* \cdot 0.640 \geq \phi_s^* \cdot 0.882$$

$$\rightarrow \phi_s^* = \frac{\phi_s}{0.882} = \frac{10}{0.882} = 11.337 \text{ mm}$$

$$\sigma_{s,tab} = \sqrt{w_k \cdot 3.48 \cdot 10^6 / \phi_s^*}$$

$$\sigma_{s,tab} = \sqrt{0.3 \cdot 3.48 \cdot 10^6 / 11.337} = 303.448 \text{ MPa}$$

$$\sigma_{s,tab} = 303.448 < \sigma_s = 405.09 \text{ MPa}$$

→ crack width control is NOT passed with given reinforcement.

(NDP) 7.3.3 Tab. 7.2DE (a)

where $w_k = 0.3 \text{ mm}$ the prescribed maximum crack width

¹The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8.

²The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

$$a_{s,req} = \frac{m_{Ed}}{\sigma_{s,tab} \cdot z} \approx \frac{m_{Ed}}{\sigma_{s,tab} \cdot 0.9 \cdot d}$$

In BEMESS the lever arm is calculated by iterating the strains ε_{top} , ε_{bot} , ε_c , ε_s . To simplify the reference $\rightarrow z = 0.9 \cdot d$ is used.

$$a_{s,req} = \frac{42.11 \cdot 100}{30.3448 \cdot 0.9 \cdot 21} = 7.34 \text{ cm}^2/\text{m}'$$

4.3 With direct calculation

Given reinforcement $a_s = 5.5 \text{ cm}^2/\text{m}'$

$$\sigma_s = \frac{m_{Ed}}{a_s \cdot z}$$

$$\sigma_s = \frac{42.11 \cdot 10^{-3}}{5.5 \cdot 10^{-4} \cdot 0.9 \cdot 0.21} = 405.09 \text{ MPa}$$

Effective area $A_{c,eff}$ of concrete in tension surrounding the reinforcement :

$$A_{c,eff} = h_{c,eff} \cdot b$$

7.3.2(3), Figure 7.1

$$A_{c,eff} = (h - d) \cdot 2 \cdot b = (24 - 21) \cdot 2 \cdot 100 = 600 \text{ cm}^2$$

$$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

Crack width w_k , 7.3.4, Eq. 7.8

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot (1 + \alpha_e \cdot \rho_{p,eff})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}$$

Crack width w_k , 7.3.4, Eq. 7.9

$$k_t = 0.4$$

7.3.4, $k_t = 0.4$ for long term loading,
 $k_t = 0.6$ for short term loading

$$\rho_{p,eff} = \frac{A_s + \xi_1^2 \cdot A'_p}{A_{c,eff}}$$

7.3.4, Eq. 7.10

$$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200000}{31476} = 6.354$$

$$A'_p = 0$$

$$\rho_{p,eff} = \frac{A_s + \xi_1^2 \cdot A'_p}{A_{c,eff}} = \frac{A_s}{A_{c,eff}}$$

7.3.4, Eq. 7.10

$$\rho_{p,eff} = \frac{5.5}{600} = 0.00916$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{405.09 - 0.4 \cdot \frac{2.56}{0.00916} \cdot (1 + 6.35 \cdot 0.00916)}{200000}$$

Crack width w_k , 7.3.4, Eq. 7.9

$$\frac{\sigma_s}{E_s} \cdot 0.6 = \frac{405.09}{200000} \cdot 0.6 = 0.00122$$

$$\varepsilon_{sm} - \varepsilon_{cm} = 0.00143 \geq 0.00122$$

(NDP) 7.3.4 (3), Eq. 7.11

$$s_{r,max} = \frac{\phi}{3.6 \cdot \rho_{p,eff}} \leq \frac{\sigma_s \cdot \phi}{3.6 \cdot f_{ct,eff}}$$

$$s_{r,max} = \frac{10}{3.6 \cdot 0.00917} \leq \frac{405.09 \cdot 10}{3.6 \cdot 2.56}$$

$$s_{r,max} = 303.03 \text{ mm} \leq 439 \text{ mm}$$

$$w_k = s_{r,max} \cdot (\epsilon_{sm} - \epsilon_{cm}) = 303.03 \cdot 0.00143$$

$$w_k = 0.43 \text{ mm} \geq w_{req} = 0.30 \text{ mm}$$

→ Check for crack is NOT passed with given reinforcement

The steps are now iterated and the reinforcement will be increased until $w_k = w_{req} = 0.3 \text{ mm}$.

Table 4: Iteration

$\alpha_s \text{ [cm}^2\text{/m]}$	$\sigma_s \text{ [MPa]}$	$w_k \text{ [mm]}$
5.5	405.0986051	0.434
5.6	397.86	0.419
5.7	390.88	0.404
5.8	384.15	0.390
5.9	377.63	0.377
6.0	371.34	0.364
...
6.6	337.58	0.300

Calculation steps for given reinforcement $\alpha_s = 6.6 \text{ cm}^2\text{/m}$

$$\sigma_s = 337.58 \text{ MPa}$$

$$A_{c,eff} = h_{c,eff} \cdot b$$

$$A_{c,eff} = (h - d) \cdot 2 \cdot b = (24 - 21) \cdot 2 \cdot 100 = 600 \text{ cm}^2$$

Crack width w_k , 7.3.4, Eq. 7.8

$$w_k = s_{r,max} \cdot (\epsilon_{sm} - \epsilon_{cm})$$

Crack width w_k , 7.3.4, Eq. 7.9

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot (1 + \alpha_e \cdot \rho_{p,eff})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}$$

7.3.4, $k_t = 0.4$ for long term loading,
 $k_t = 0.6$ for short term loading

$$k_t = 0.4$$

7.3.4, Eq. 7.10

$$\rho_{p,eff} = \frac{A_s + \xi_1^2 \cdot A'_p}{A_{c,eff}}$$

7.3.4, α_e is the ratio E_s/E_{cm}

$$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200000}{31476} = 6.354$$

$$A'_p = 0$$

$$\rho_{p,eff} = \frac{A_s + \xi_1^2 \cdot A'_p}{A_{c,eff}} = \frac{A_s}{A_{c,eff}}$$

$$\rho_{p,eff} = \frac{6.6}{600} = 0.0110$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{337.58 - 0.4 \cdot \frac{2.56}{0.0110} \cdot (1 + 6.35 \cdot 0.0110)}{200000} \quad 7.3.4, \text{ Eq. 7.9}$$

$$\frac{\sigma_s}{E_s} \cdot 0.6 = \frac{337.58}{200000} \cdot 0.6 = 0.00101$$

$$\varepsilon_{sm} - \varepsilon_{cm} = 0.0011899 \geq 0.00101$$

$$s_{r,max} = \frac{\phi}{3.6 \cdot \rho_{p,eff}} \leq \frac{\sigma_s \cdot \phi}{3.6 \cdot f_{ct,eff}} \quad (\text{NDP}) 7.3.4 (3), \text{ Eq. 7.11}$$

$$s_{r,max} = \frac{10}{3.6 \cdot 0.0110} \leq \frac{337.58 \cdot 10}{3.6 \cdot 2.56}$$

$$s_{r,max} = 252.52 \text{ mm} \leq 366.30 \text{ mm}$$

$$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm}) = 252.52 \cdot 0.0011899$$

$$w_k = 0.30 \text{ mm} \leq w_{req} = 0.30 \text{ mm}$$

→ Check for crack is passed with given reinforcement

5 Conclusion

This example shows the calculation of crack widths. Various ways of reference calculations are demonstrated, in order to compare the SOFiSTiK results to. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

- [1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010.* CEN. 2010.
 - [2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau.* BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.
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