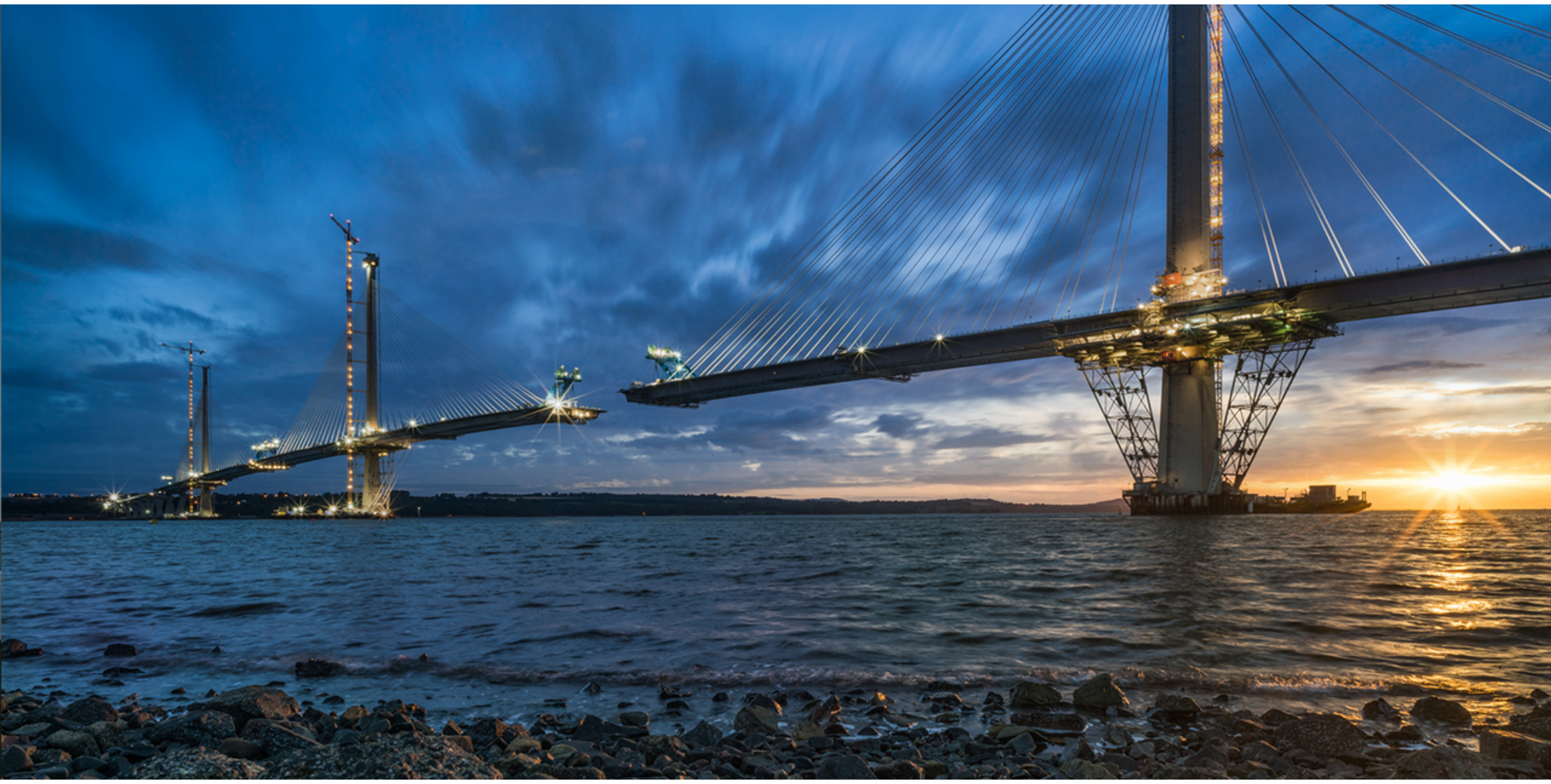

Benchmark Example No. 2

Design of a Rectangular CS for Bending

SOFiSTiK | 2020

VERiFiCATiON
DCE-EN2 Design of a Rectangular CS for Bending

VERiFiCATiON Manual, Service Pack 2020-14 Build 134

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Front Cover

Project: Queensferry Crossing | Photo: Bastian Kratzke

Overview

Design Code Family(s): DIN
Design Code(s): DIN EN 1992-1-1
Module(s): AQB
Input file(s): rectangular_bending.dat

1 Problem Description

The problem consists of a rectangular section, as shown in Fig. 1. The cross-section is designed for an ultimate moment M_{Ed} and the required reinforcement is determined.

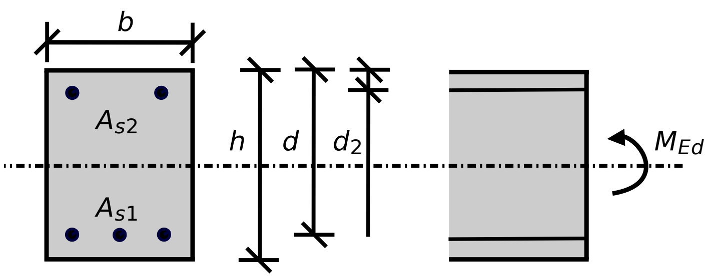

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the design of doubly reinforced sections for ULS, subject to pure flexure, such as beams. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Basic assumptions for section design (Section 6.1)
- Reinforcement (Section 9.3.1.1, 9.2.1.1)

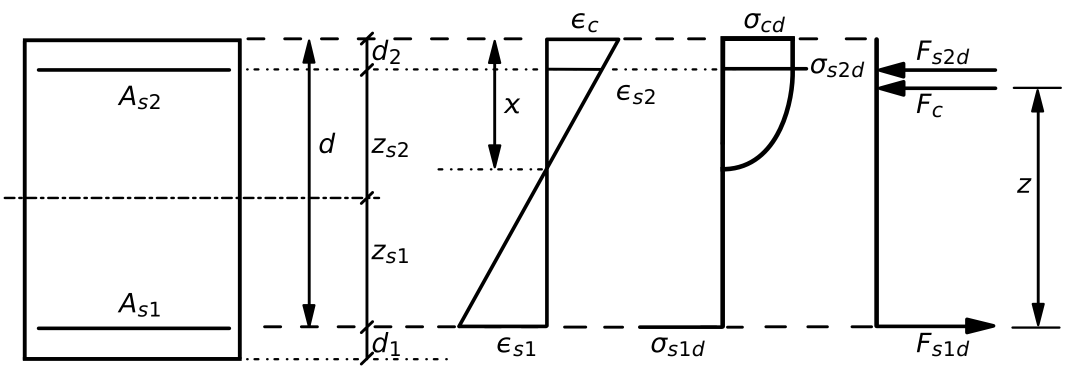

Figure 2: Stress and Strain Distributions in the Design of Doubly Reinforced Cross-sections

In doubly reinforced rectangular beams, the conditions in the cross-section at the ultimate limit state, are assumed to be as shown in Fig. 2. The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).

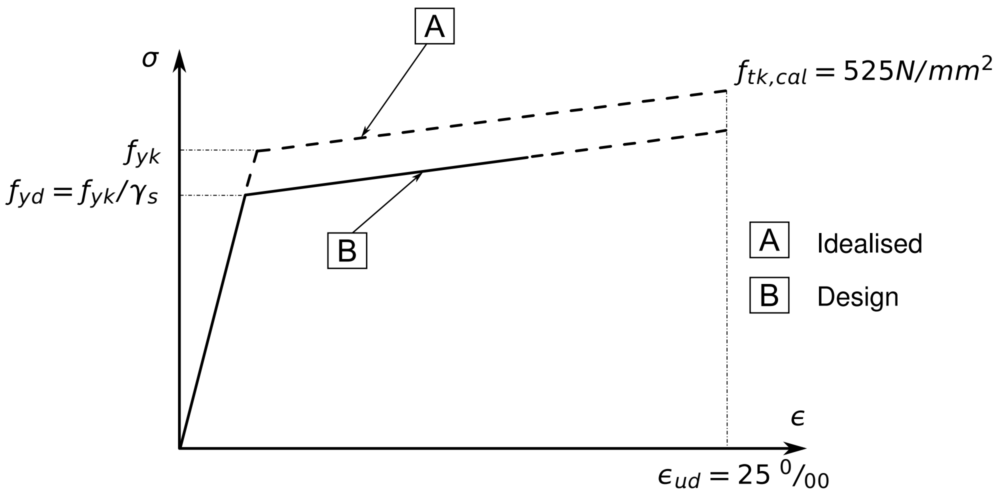

Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The rectangular cross- section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], to carry an ultimate moment of $135\ kNm$. The calculation steps with different design methods [3] [4] [5] are presented below and the results are given in Table 2. Here, it has to be mentioned that these standard methods employed in order to calculate the reinforcement are approximate, and therefore deviations often occur.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\,20/25$	$h = 40.0\,cm$	$M_{Ed} = 135\,kNm$
$B\,500A$	$d = 35.0\,cm$	
	$d_2 = 5.0\,cm$	
	$b = 25\,cm$	

Table 2: Results

	SOF.	General Chart [3]	ω—Table [3]	k_d—Table [3]
$A_{s1}\ [cm^2/m]$	10.73	10.73	10.77	10.79
$A_{s2}\ [cm^2/m]$	2.47	2.47	2.52	2.43

4 Design Process[1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

Concrete: $\gamma_c = 1.50$

Steel: $\gamma_s = 1.15$

(NDP) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

$f_{ck} = 20\ MPa$
$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 0.85 \cdot 20/1.5 = 11.33\ MPa$

Tab. 3.1: Strength for concrete

3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{yk} = 500\ MPa$
$f_{yd} = f_{yk}/\gamma_s = 500/1.15 = 434.78\ MPa$

3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$

3.2.7: (2), Fig. 3.8

Design Load:

$N_{Ed} = 0$
$M_{Eds} = M_{Ed} - N_{Ed} \cdot z_{s1} = 135\ kNm$

Design with respect to General Design Chart Bending with axial force for rectangular cross-sections:

$$\mu_{Eds} = \frac{M_{Eds}}{b \cdot d^2 \cdot f_{cd}} = \frac{135 \cdot 10^{-3}}{0.25 \cdot 0.35^2 \cdot 11.33} = 0.389$$

$\mu_{Eds} > \mu_{Eds,lim} = 0.296$

5.4: (NA.5): Linear elastic analysis

ξ = height of comression zone $x/d \leq 0.45$ for $C12/15 - C50/60$

→ *compression reinforcement required*

from design chart for $\mu_{Eds,lim} = 0.296$ and $d_2/d = 0.143$:
$\epsilon_{s1} = 4.30 \cdot 10^{-3}$; $\epsilon_{s2} = -2.35 \cdot 10^{-3}$; $\zeta = z/d = 0.813$

Tab. 9.1 [3]: General Chart for up to $C50/60$ - Section with compression reinforcement

for $\epsilon_{s1} = 4.30 \cdot 10^{-3} \rightarrow \sigma_{s1d} = 436.8\ MPa$
for $\epsilon_{s2} = -2.35 \cdot 10^{-3} \rightarrow \sigma_{s1d} = -434.9\ MPa$

$M_{Eds,lim} = \mu_{Eds,lim} \cdot b \cdot d^2 \cdot f_{cd} = 102.7\ kNm$
$\Delta M_{Eds} = M_{Eds} - M_{Eds,lim} = 135 - 102.7 = 32.3\ kNm$

$$A_{s1} = \frac{1}{\sigma_{s1d}} \cdot \left(\frac{M_{Eds,lim}}{\zeta \cdot d} + \frac{\Delta M_{Eds}}{d - d_2} + N_{Ed} \right) = 10.73\ cm^2$$

$$A_{s2} = \frac{1}{|\sigma_{s2d}|} \cdot \frac{\Delta M_{Eds}}{d - d_2} = 2.47\ cm^2$$

Design with respect to $\omega-$ (or μ_s-)Table for rectangular cross-sections:

$$\mu_{Eds} = \frac{M_{Eds}}{b \cdot d^2 \cdot f_{cd}} = \frac{135 \cdot 10^{-3}}{0.25 \cdot 0.35^2 \cdot 11.33} = 0.389$$

5.4: (NA.5): Linear elastic analysis

ξ = height of comression zone $x/d \leq 0.45$ for $C12/15 - C50/60$

Because the internal force determination is done on the basis of a linear

[1]The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

[2]The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

Tab. 9.2 [3]: ω—Table for up to $C50/60$ - Rectangular section with compression reinforcement

elastic calculation, then $\xi_{lim} = 0.45$ is chosen. Referring to the design table with compression reinforcement and for $d_2/d = 0.15$:

$\omega_1 = 0.4726; \quad \omega_1 = 0.1104$

$$A_{s1} = \frac{1}{f_{yd}} \cdot (\omega_1 \cdot b \cdot d \cdot f_{cd} + N_{Ed}) = 10.77\ cm^2$$

$$A_{s2} = \frac{f_{cd}}{f_{yd}} \cdot (\omega_2 \cdot b \cdot d) = 2.52\ cm^2$$

Design with respect to k_d— Design Table for rectangular cross-sections:

$$k_d = \frac{d}{\sqrt{M_{Eds}/b}} = \frac{35}{\sqrt{135/0.25}} = 1.51$$

Tab. 9.3 [3]: k_d—Table for up to $C50/60$ - Rectangular section with compression reinforcement

Not able to read values from k_d—table for simply reinforced rectangular cross-sections
→ *compression reinforcement is required*

5.4: (NA.5): Linear elastic analysis
ξ = height of comression zone $x/d \leq 0.45$ for $C12/15 - C50/60$

Because the internal force determination is done on the basis of a linear elastic calculation, then $\xi_{lim} = 0.45$ is chosen. Referring to the k_d—table with compression reinforcement:

$k_{s1} = 2.740; \quad k_{s2} = 0.575$
(interpolated values for $k_d = 1.51$)

$\rho_1 = 1.021; \quad \rho_2 = 1.097$
(interpolated values for $d_2/d = 0.143$ and $k_{s1} = 2.740$)

$$A_{s1} = \rho_1 \cdot k_{s1} \cdot \frac{M_{Eds}}{d} + \frac{N_{Ed}}{\sigma_{s1d}} = 10.79\ cm^2$$

$$A_{s2} = \rho_2 \cdot k_{s2} \cdot \frac{M_{Eds}}{d} = 2.43\ cm^2$$

5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular beam cross-section under bending. Various different reference solutions are employed in order to compare the SOFiSTiK results to. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] K. Holschemacher, T. Müller, and F. Lobisch. *Bemessungshilfsmittel für Betonbauteile nach Eurocode 2 Bauingenieure*. 3rd. Ernst & Sohn, 2012.

[4] *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau*. Ernst & Sohn. Deutschen Beton- und Bautechnik-Verein E.V. 2011.

[5] R. S. Narayanan and A. W. Beeby. *Designers' Guide to EN 1992-1-1 and EN 1992-1-2 - Eurocode 2: Design of Concrete Structures*. Thomas Telford, 2005.