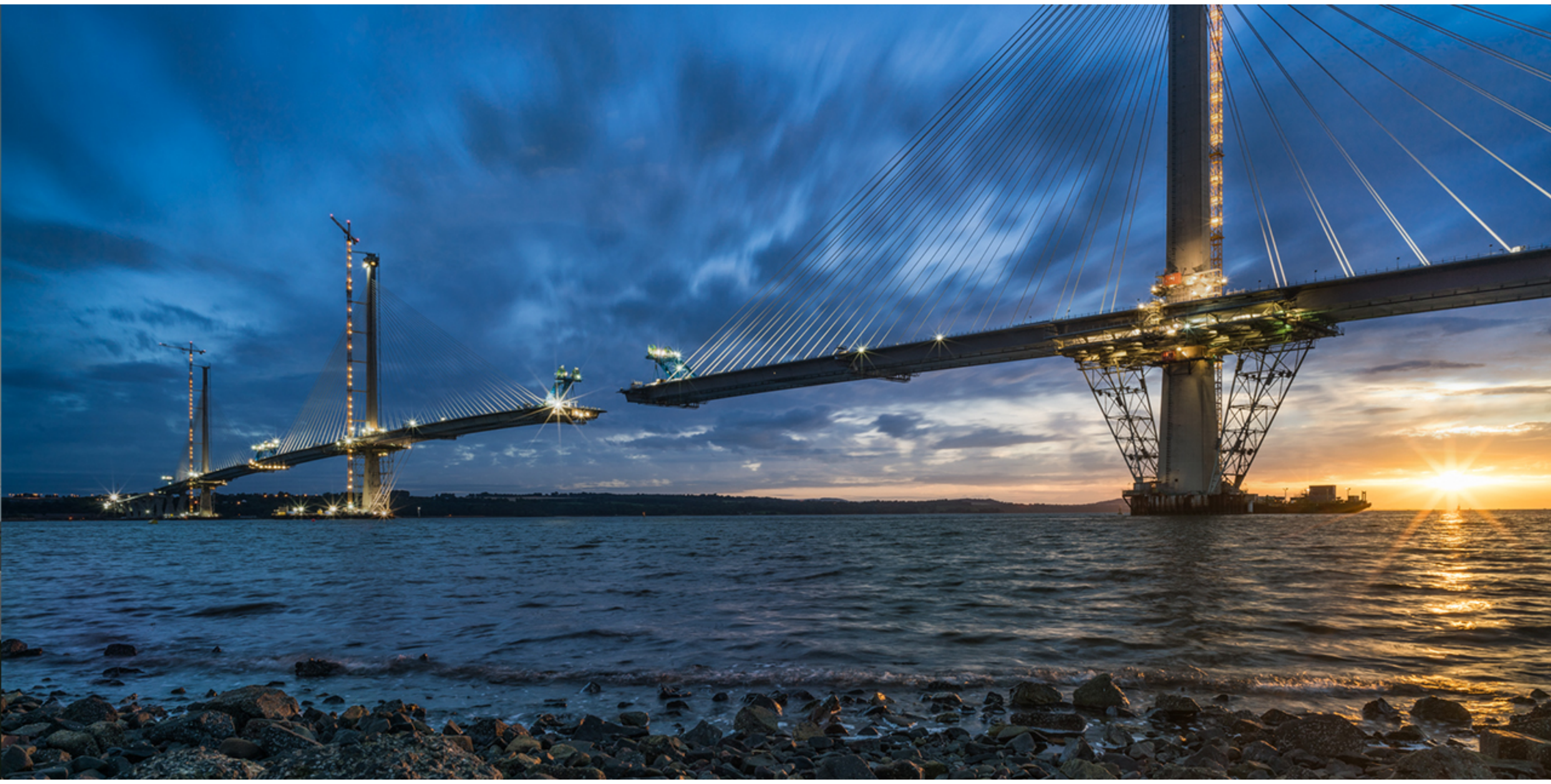

Benchmark Example No. 34

Elastic Critical Plate Buckling Stress

SOFiSTiK | 2020

VERiFiCATiON
DCE-EN34 Elastic Critical Plate Buckling Stress

VERiFiCATiON Manual, Service Pack 2020-14 Build 134

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Front Cover

Project: Queensferry Crossing | Photo: Bastian Kratzke

Overview	
Design Code Family(s):	EN
Design Code(s):	EN 1993-1-1
Module(s):	ASE
Input file(s):	eccs_plate_buckling.dat

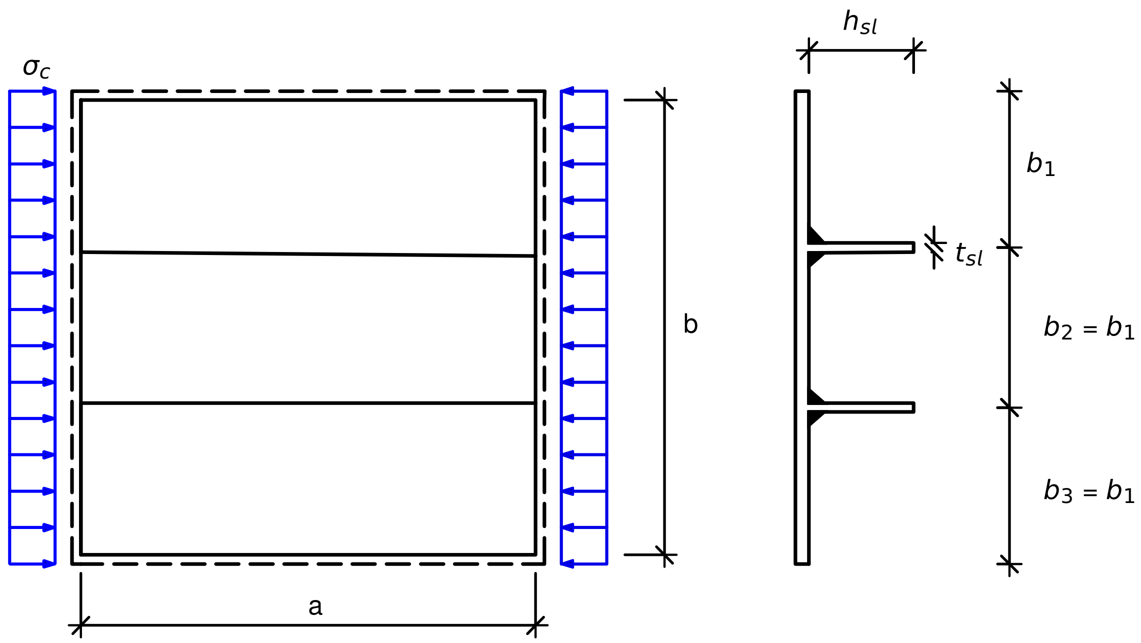
1 Problem Description

The problem consists of a stiffened steel plate. Its dimensions and boundary conditions are given in Figure 2.

The main goal of this benchmark is to verify and compare the SOFiSTiK results with the ECCS reference example Beg et al. [1, Example 2.4-3].

In SOFiSTiK a FEM model will be used to compare the results with:

- Klöppel diagrams (Klöppel and Scheer, 1960)
- EBPlate (2007)
- FEM software (ABAQUS)
- EN 1993-1-5 rules



a = 1800 mm, b = 1800 mm, b_1 = 600 mm, h_{sl} = 100 mm, t_{sl} = 10 mm.
S 355, $f_y = 355\ N/mm^2$, $\varepsilon = 0.81$

Figure 1: The layout of stiffened plate

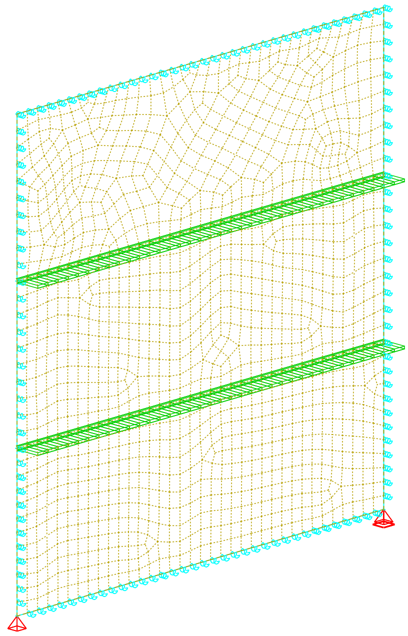


Figure 2: SOFiSTiK FEM Model - Steel plate with 2 horizontal stiffeners

2 Reference Solution

This example is concerned with calculation of the elastic critical plate buckling stress. The content of this problem is covered by following parts of EN 1993-1-1 [2] and EN 1993-1-5 [3]:

- Materials (EN 1993-1-1 [2], Section 3)
- Calculating the critical plate buckling stress (EN 1993-1-5 [3], Annex A.2)

3 Model and Results

The calculation steps with loading conditions are presented below and the results are given in Table 2.

To calculate the critical plate buckling stress the loading, $\sigma_c = 1.0\ N/mm^2$ is used. The critical elastic stress will be calculated by multiplying the minimum eigenvalue with the unity stress $\sigma_c = 1.0$.

$\sigma_{cr,p} = \alpha_{cr} \cdot \sigma_c$

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$S\ 355$	$a = 1800\ mm$	$\sigma_c = -1\ N/mm^2$
$f_y = 355\ N/mm^2$	$b = 1800\ mm$	
$\varepsilon = 0.81$	$b_1 = 600\ mm$	
	$h_{sl} = 100\ mm$	
	$t_{sl} = 10\ mm$	
	$t = 12\ mm$	

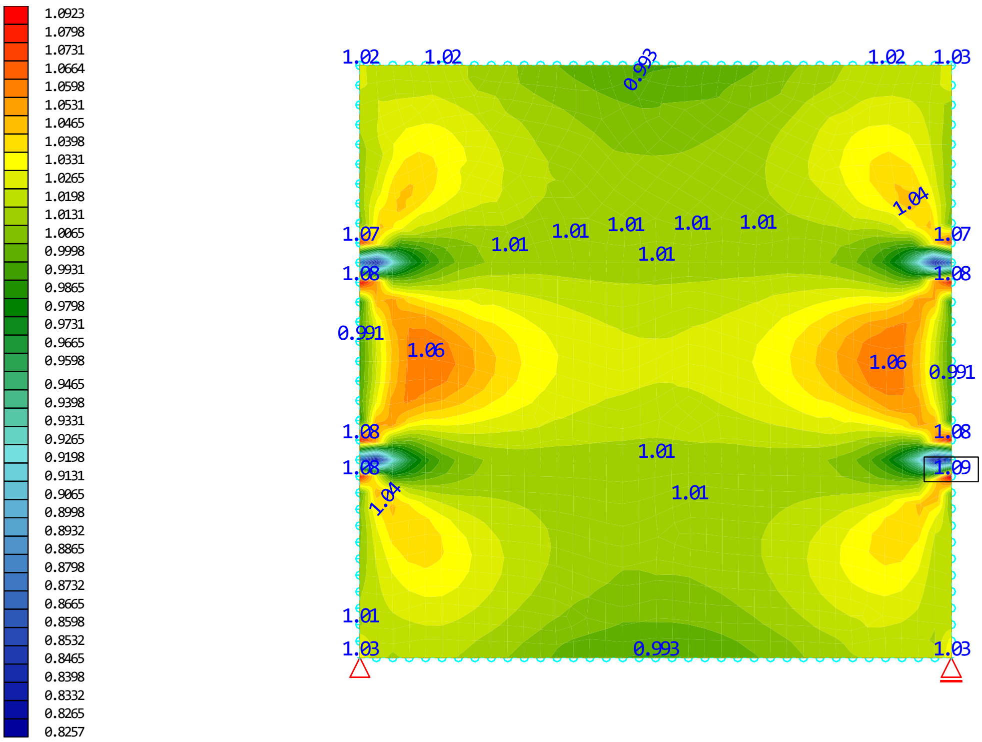


Figure 3: SOFiSTiK FEM Model - Stress distribution along the plate for $\sigma_c = 1.0\ N/mm^2$

Table 2: Results

	Units	SOF.	Klöppel	EBPlate	ABAQUS	EN 1993-1-5
σ_{cr}	$\left[\frac{N}{mm^2}\right]$	275.782	274.170	268.72[1]	268[2]	290

Table 3: SOFiSTiK Buckling Eigenvalues

No.	Loadcase	Relative Error	Buckling Factor
1.	2001	2.03E-21	**275.782**
2.	2002	4.65E-16	324.372
3.	2003	3.77E-11	350.856
4.	2004	8.56E-10	361.965
5.	2005	1.38E-09	375.972

[1] EBPlate, V2.01
[2] The results were overtaken from the ECCs reference example Beg et al. [1, Example 2.4-3]

4 Design Process

Design with respect to EN 1993-1-5:2006 [3]: [3]

4.1 Klöppel

$\sigma_{cr,p}$ is given with the following equation:

$$\sigma_{cr,p} = k_{\sigma,p} \cdot \sigma_E$$

Where:

$$\sigma_E = \frac{\pi^2 \cdot E \cdot t^2}{12 \cdot (1 - \nu^2) \cdot b^2}$$

$$\sigma_E = \frac{\pi^2 \cdot 210000 \cdot 12^2}{12 \cdot (1 - 0.3^2) \cdot 1800^2} = 8.436\ N/mm^2$$

$k_{\sigma,p}$ is the elastic critical plate buckling coefficient according to Klöppel.

The parameters needed for the evaluation of $k_{\sigma,p}$ are:

$$\alpha = \frac{a}{b} = \frac{1800}{1800} = 1.0$$

$$\delta = \frac{A_{sl}}{b \cdot t} = \frac{b_{sl} \cdot t}{b \cdot t} = \frac{100 \cdot 10}{1800 \cdot 12} = 0.05$$

$$\gamma = \frac{(I_{sl} + A_{sl} \cdot e^2) \cdot 12 \cdot (1 - \nu^2)}{b \cdot t^3}$$

$$\gamma = \frac{\left(b_{sl}^3 \cdot \frac{t_{sl}}{12} + b_{sl} \cdot t_{sl} \cdot e^2\right) \cdot 12 \cdot (1 - \nu^2)}{b \cdot t^3}$$

$$\gamma = \frac{\left(100^3 \cdot \frac{10}{12} + 100 \cdot 10 \cdot 50^2\right) \cdot 12 \cdot \left(1 - 0.3^2\right)}{1800 \cdot 12^3} = 11.70$$

Note that parameter α and δ above are not the same as in EN 1993-1-5, Annex A.1, where the procedure for plates stiffened with more than two stiffeners is given.

The plate buckling coefficient is obtained from the diagram (according to Klöppel) in Figure 4.

$$k_\sigma = 32.5$$

Finally, the critical buckling stress is equal to:

$$\sigma_{cr,p} = k_\sigma \cdot \sigma_E = 32.5 \cdot 8.436 = 274.17\ N/mm^2$$

4.2 EBPlate

The usual procedure (calculation of buckling modes) for the calculation of critical stresses is presented by using EBPlate.

[3] The sections mentioned in the margins refer to EN 1993-1-5:2006 [3] unless otherwise specified.

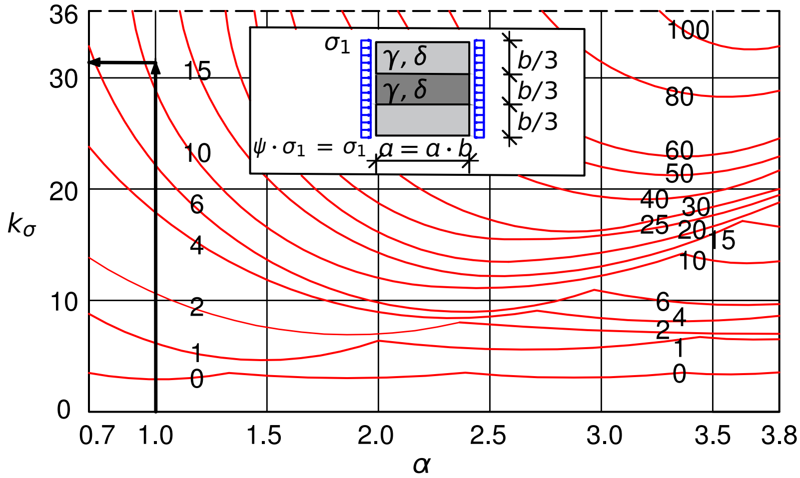


Figure 4: Klöppel Diagram

plate: $a = 1800\ mm$, $b = 1800\ mm$, $t = 12\ mm$

stiffener: $h = b_{sl} = 100\ mm$, $t = t_{sl} = 10\ mm$

stiffener position: $b_1 = 600\ mm$

Results (1st and 2nd buckling mode)

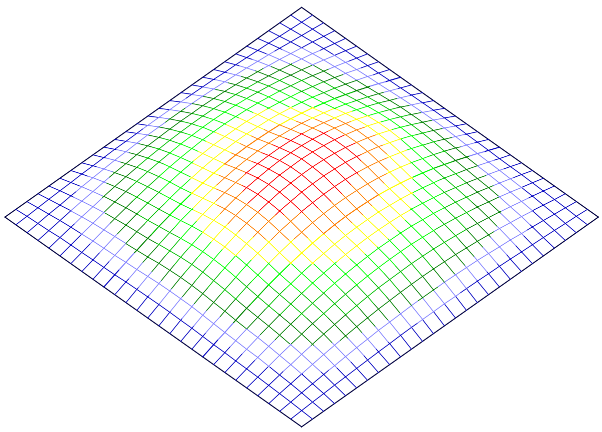


Figure 5: EBPlate - Buckling mode 1, $\sigma_{cr,p} = 268.72\ N/mm^2$

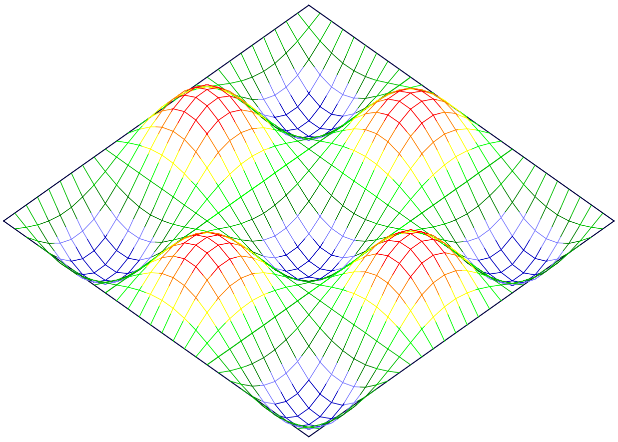
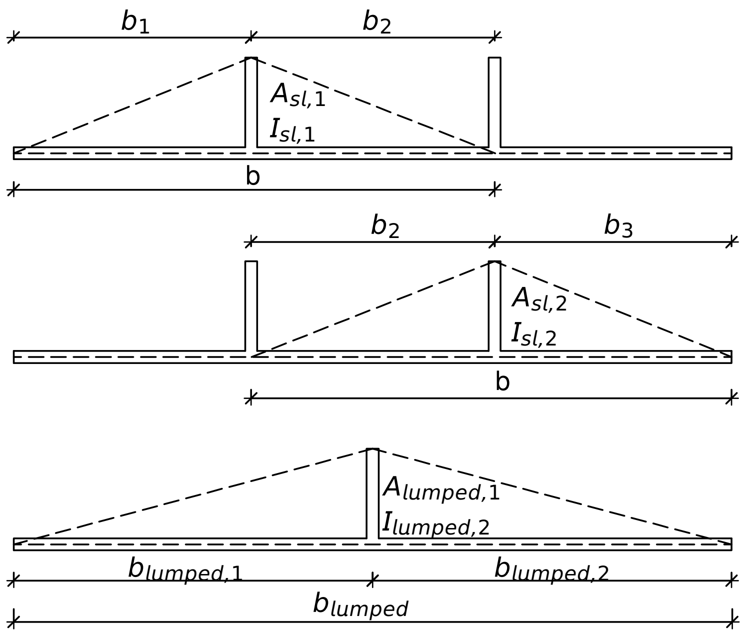


Figure 6: EBPlate - Buckling mode 2, $\sigma_{cr,p} = 316.508\ N/mm^2$

4.3 EN 1993-1-5, Annex A.2

The critical plate buckling stress is calculated according to EN 1993-1-5, Annex A.2. The plate can be treated as an equivalent orthotropic plate if it is stiffened with at least three stiffeners. The plate-like behaviour is modelled by the buckling of each stiffener as a column on continuous elastic support provided by plate, while the other stiffeners acts as rigid support. Buckling of both stiffeners simultaneously is accounted for by considering a single lumped stiffener, which substitutes both stiffeners in such a way that its cross-sectional area and its second moment of area are the sum of the individual stiffeners. It is positioned at the location of the resultant of the respective forces in the individual stiffeners.



Stiffeners I and II

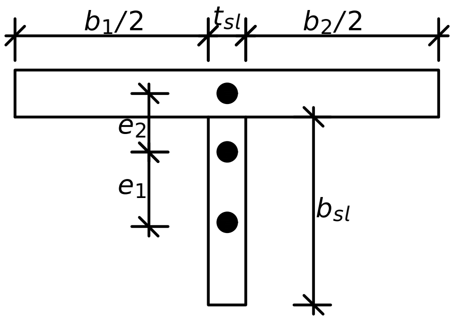


Figure 7

$e_1 = 49.20\ mm$, $e_2 = 6.80\ mm$, $\overline{b}_1 = 595\ mm$, $\overline{b}_2 = 590\ mm$

$b_1 = b_2 = 600\ mm$, $b = b_1 + b_2 = 600 + 600 = 1200\ mm$

$$A_{sl,1} = \left(\frac{\overline{b}_1 + \overline{b}_2}{2} + t_{sl}\right) \cdot t + b_{sl} \cdot t_{sl}$$

$$A_{sl,1} = \left(\frac{595 + 590}{2} + 10\right) \cdot 12 + 100 \cdot 10 = 8230\ mm^2$$

$$I_{sl,1} = \frac{b_{sl}^3 \cdot t_{sl}}{12} + \frac{\left((\overline{b}_1 + \overline{b}_2) \cdot 0.5 + t_{sl,1}\right) \cdot t^3}{12} + b_{sl} \cdot t_{sl} \cdot e_1^2 + \left((\overline{b}_1 + \overline{b}_2) \cdot 0.5 + t_{sl,1}\right) \cdot t \cdot e_2^2$$

$$I_{sl,1} = \frac{100^3 \cdot 10}{12} + \frac{((595 + 590) \cdot 0.5 + 10) \cdot 12}{12} + 100 \cdot 10 \cdot 49.2^2 + ((595 + 590) \cdot 0.5 + 10) \cdot 12 \cdot 6.80^2$$

$$I_{sl,1} = 3.68 \cdot 10^6\ mm^4$$

$$a_c = 4.33 \cdot \sqrt[4]{\frac{I_{sl,1} \cdot b_1^2 \cdot b_2^2}{t^3 \cdot b}}$$

$$a_c = 4.33 \cdot \sqrt[4]{\frac{3.68 \cdot 10^6 \cdot 600^2 \cdot 600^2}{12^3 \cdot 1200}}$$

$$a_c = 2998\ mm$$

As $a \leq a_c$ ($a = 1800\ mm$), the column buckles in a 1-wave mode and the buckling stress is obtained as follows:

$$\sigma_{cr,sl} = \frac{\pi^2 \cdot E \cdot I_{sl,1}}{A_{sl,1} \cdot a^2} + \frac{E \cdot t^3 \cdot b \cdot a^2}{4 \cdot \pi^2 \cdot (1-\nu^2) \cdot A_{sl,1} \cdot b_1^2 \cdot b_2^2}$$

$$\sigma_{cr,sl} = \frac{\pi^2 \cdot 210000 \cdot 3.68 \cdot 10^6}{8230 \cdot 1800^2} + \frac{210000 \cdot 12^3 \cdot 1200 \cdot 1800^2}{4 \cdot \pi^2 \cdot (1-0.3^2) \cdot 8230 \cdot 600^2 \cdot 600^2}$$

$$\sigma_{cr,sl} = 322\ N/mm^2$$

In case of a stress gradient over the plate width, the critical plate buckling stress should be properly interpolated from the position of the stiffener to the most stressed edge of the plate. In this case no stress gradient over the depth of the plate is present. Therefore, the critical plate buckling stress is equal to the critical stress calculated for the buckling of the stiffener on the elastic support:

$$\sigma^I_{cr,p} = \sigma^{II}_{cr,p} = \sigma_{cr,sl} = 322\ N/mm^2$$

Lumped stiffener

$$b_{lumped,1} = b_{lumped,2} = 900\ mm,\ b_{lumped} = 1800\ mm$$

$$A_{lumped} = A^I_{sl} + A^{II}_{sl} = 8230 + 8230 = 16460\ mm^4$$

$$I_{lumped} = I^I_{sl} + I^{II}_{sl} = 3.675 \cdot 10^6 + 3.675 \cdot 10^6 = 7.35 \cdot 10^6\ mm^4$$

$$a_c = 4.33 \cdot \sqrt[4]{\frac{I_{lumped} \cdot b^2_{lumped,1} \cdot b^2_{lumped,2}}{t^3 \cdot b_{lumped}}}$$

$$a_c = 4.33 \cdot \sqrt[4]{\frac{7.35 \cdot 10^6 \cdot 900^2 \cdot 900^2}{12^3 \cdot 1800}}$$

$$a_c = 4832\ mm$$

As $a < a_c$ ($a = 1800\ mm$), the column buckles in a 1-wave mode and the buckling stress is obtained with equation:

$$\sigma_{cr,lumped} = \frac{\pi^2 \cdot E \cdot I_{lumped}}{A_{lumped} \cdot a^2} + \frac{E \cdot t^3 \cdot b_{lumped} \cdot a^2}{4 \cdot \pi^2 \cdot (1-\nu^2) \cdot A_{lumped} \cdot b^2_{lumped,1} \cdot b^2_{lumped,2}}$$

$$\sigma_{cr,lumped} = \frac{\pi^2 \cdot 210000 \cdot 7.35 \cdot 10^6}{16460 \cdot 1800^2} + \frac{210000 \cdot 12^3 \cdot 1800 \cdot 1800^2}{4 \cdot \pi^2 \cdot (1 - 0.3^2) \cdot 16460 \cdot 900^2 \cdot 900^2}$$

Finally we have:

$$\sigma_{cr,lumped} = 290\ N/mm^2$$

$$\sigma_{cr,p} = min\left[\sigma^I_{cr,p}, \sigma^{lumped}_{cr,p}\right]$$

$$\sigma_{cr,p} = min\,[322, 290] = 290\ N/mm^2$$

4.4 ABAQUS

The results from ABAQUS have be overtaken from the ECCS reference example Beg et al. [1, Example 2.4-3], see Table 2 for more details.

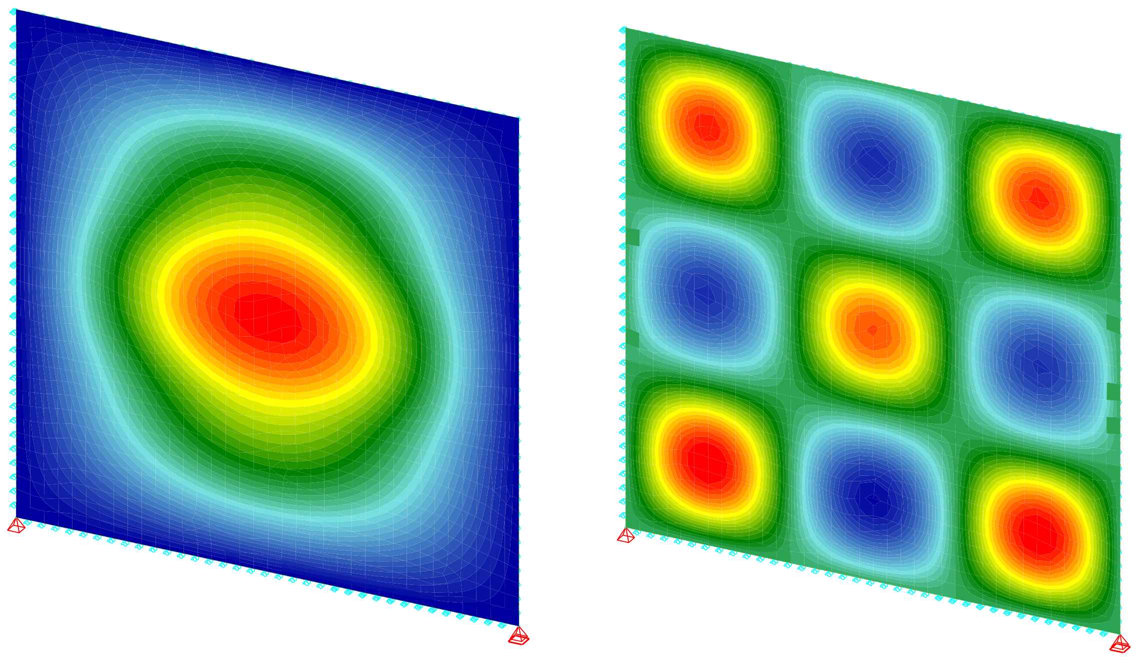
5 Conclusion

The critical plate buckling stress was calculated by using:

- SOFiSTiK
- Klöppel diagrams
- EBPlate
- ABAQUS
- EN 1993-1-5 rules

All results are compared and summarised in Table 2.

The methods used in the calculation give very similar results. The advantage of SOFiSTiK compared to Eurocode formulas and other tools (that are not using FEM) is that the stiffeners can be added customly. The cases with variable height of plates can be analysed as well (haunches). In conclusion, it has been shown that the SOFISTiK results are reproduced with excellent accuracy.



(a) 1st buckling mode: $\sigma_{cr,p} = 275.782\ N/mm^2$ (b) 2nd buckling mode: $\sigma_{cr,p} = 324.372\ N/mm^2$

Figure 8: SOFiSTiK - Buckling modes

6 Literature

[1] D. Beg et al. *Design of plated structures*. Ernst & Sohn and ECCS, 2010.

[2] *EN 1993-1-1: Eurocode 3: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2005.

[3] *EN 1993-1-5: Eurocode 3: Design of steel structures, Part 1-5: Plated structural elements*. CEN. 2006.