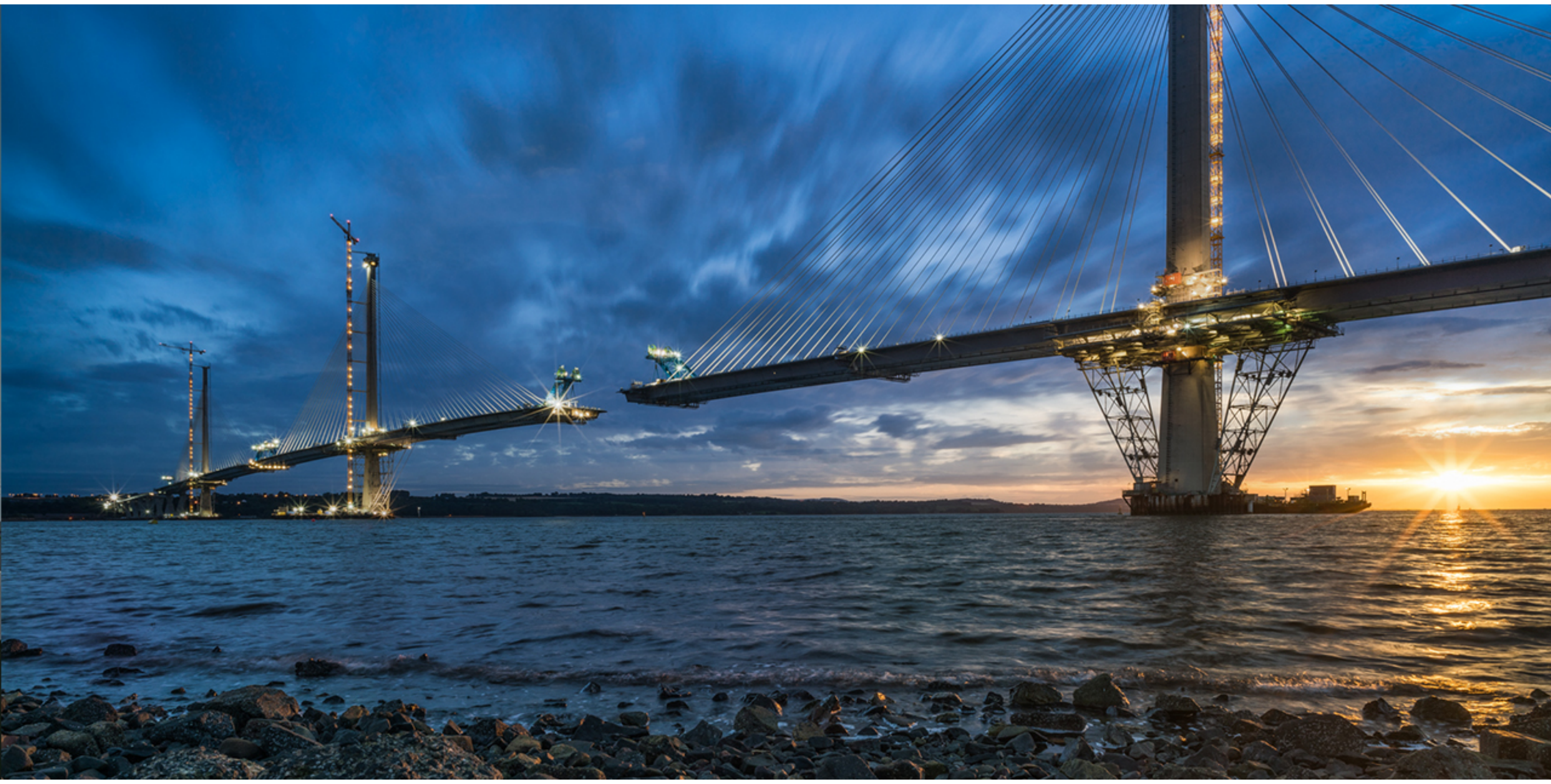

Benchmark Example No. 22

Stress Relaxation of Prestressing Steel - EN 1992-1-1

SOFiSTiK | 2020

VERiFiCATiON
DCE-EN22 Stress Relaxation of Prestressing Steel - EN 1992-1-1

VERiFiCATiON Manual, Service Pack 2020-14 Build 134

SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

Front Cover

Project: Queensferry Crossing | Photo: Bastian Kratzke

Overview	
Design Code Family(s):	EN
Design Code(s):	EN 1992-1-1
Module(s):	AQB, AQUA
Input file(s):	relaxation_en1992.dat

1 Problem Description

The problem consists of a simply supported beam with a rectangular cross-section of prestressed concrete, as shown in Fig. 1. The time dependent losses are calculated, considering the reduction of stress caused by the deformation of prestressing steel due to steel-relaxation, under the permanent loads.

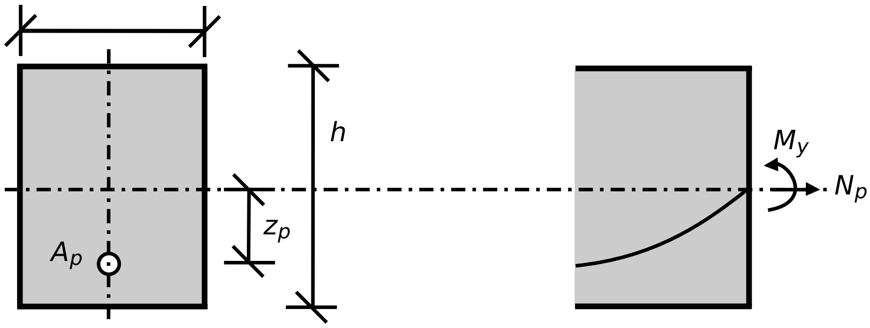

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the calculation of relaxation losses on a prestressed concrete cs, subject to bending and prestress force. The content of this problem is covered by the following parts of EN 1992-1-1:2004 [1]:

- Properties (Section $3.3.2$)
- Annex D: Detailed calculation method for prestressing steel relaxation losses (Section $D.1$)
- Strength (Section $3.3.3$)

In this Benchmark the stress loss due to relaxation will be examined, creep and shrinkage losses are neglected and disabled.

3 Model and Results

Benchmark 17 is here extended for the case of steel relaxation losses developing on a prestressed concrete simply supported beam. The analysed system can be seen in Fig. 2, with properties as defined in Table 1. Further information about the tendon geometry and prestressing can be found in benchmark 17. The beam consists of a rectangular cs and is prestressed and loaded with its own weight. A calculation of the relaxation stress losses is performed in the middle of the span with respect to EN 1992-1-1:2004 [1]. The calculation steps are presented below and the results are given in Table 2 for the calculation with AQB.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading (at $x = 10\ m$)	Time
$C\ 35/45$	$h = 100.0\ cm$	$M_g = 1250\ kNm$	$t = 1000\ h$
$Y\ 1770$	$b = 100.0\ cm$	$N_p = -3653.0\ kN$	
$RH = 80$	$L = 20.0\ m$		
	$A_p = 28.5\ cm^2$		

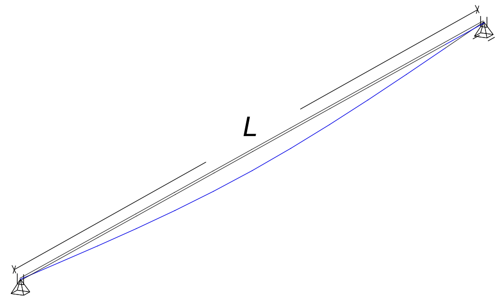

Figure 2: Simply Supported Beam

Table 2: Results

Result	AQB	Ref.
$\Delta\sigma_{pr,total}$	$14.38\ MPa$	$14.38\ MPa$
ΔP_{pr}	$40.95\ kN$	$40.95\ kN$
$\frac{\Delta P_{pr}}{P_0}[\%]$	1.12%	1.12%

4 Design Process[1]

Design with respect to EN 1992-1-1:2004 [1]

• Material:

Concrete: $C\ 35/45$

3.1: Concrete

$E_{cm} = 34077\ N/mm^2$

3.1.2: Tab. 3.1: E_{cm}, f_{ck} and f_{cm} for $C\ 35/45$

$f_{ck} = 35\ N/mm^2$

$f_{cm} = 43\ N/mm^2$

Prestressing Steel: $Y\ 1770$

3.3: Prestressing Steel

$E_p = 195000\ N/mm^2$

3.3.6 (3): E_p for wires

$f_{pk} = 1770\ N/mm^2$

3.3.2, 3.3.3: f_{pk} Characteristic tensile strength of prestressing steel

$f_{p0,1k} = 1520\ N/mm^2$

3.3.2, 3.3.3: $f_{p0,1k}$ 0.1% proof-stress of prestressing steel, yield strength

Prestressing system: $BBV\ L19\ 150\ mm^2$

19 wires with area of $150\ mm^2$ each, giving a total of $A_p = 28.5\ cm^2$

• Cross-section:

$A_c = 1.0 \cdot 1.0 = 1\ m^2$

Diameter of duct $\phi_{duct} = 97\ mm$

Ratio $\alpha_{E,p} = E_p\ /\ E_{cm} = 195000\ /\ 34077 = 5.7223$

$A_{c,netto} = A_c - \pi \cdot (\phi_{duct}/2)^2 = 0.9926\ m^2$

$A_{ideal} = A_c + A_p \cdot \alpha_{E,p} = 1.013\ m^2$

• Prestressing forces and stresses

The force applied to a tendon, i.e. the force at the active end during tensioning, should not exceed the following value

$P_{max} = A_p \cdot \sigma_{p,max}$

where $\sigma_{p,max} = min(0.8 \cdot f_{pk}; 0.90 \cdot f_{p0,1k})$

$P_{max} = A_p \cdot 0.80 \cdot f_{pk} = 28.5 \cdot 10^{-4} \cdot 0.80 \cdot 1770 = 4035.6\ kN$

$P_{max} = A_p \cdot 0.90 \cdot f_{p0,1k} = 28.5 \cdot 10^{-4} \cdot 0.90 \cdot 1520 = 3898.8\ kN$

$\rightarrow P_{max} = 3898.8\ kN$ and $\sigma_{p,max} = 1368\ N/mm^2$

The value of the initial prestress force at time $t = t_0$ applied to the concrete immediately after tensioning and anchoring should not exceed the following value

5.10.3 (2): Prestress force

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.3.6: Fig. 3.10

5.10.3 (2): Eq. 5.43: P_{m0} initial prestress force at time $t = t_0$

(NDP) 5.10.3 (2): $\sigma_{p,m0}(x)$ stress in the tendon immediately after tensioning or transfer

$$P_{m0}(x) = A_p \cdot \sigma_{p,m0}(x)$$

where $\sigma_{p,m0}(x) = \min\left\{0.75 f_{pk}; 0.85 f_{p0,1k}\right\}$

$$P_{m0} = A_p \cdot 0.75 \cdot f_{pk} = 28.5 \cdot 10^{-4} \cdot 0.75 \cdot 1770 = 3783.4\ kN$$

$$P_{m0} = A_p \cdot 0.85 \cdot f_{p0,1k} = 28.5 \cdot 10^{-4} \cdot 0.85 \cdot 1520 = 3682.2\ kN$$

$\rightarrow P_{m0} = 3682.2\ kN$ and $\sigma_{p,m0} = 1292\ N/mm^2$

Further calculations for the distribution of prestress forces and stresses along the beam are not in the scope of this benchmark and will not be described here. The complete diagram can be seen in benchmark 17, after the consideration of losses at anchorage and due to friction, as calculated by SOFiSTiK. There the values of $\sigma_{p,max} = 1368\ N/mm^2$ and $P_{m0} = 3682.2\ kN$ can be visualised.

In this benchmark the beam number 10010 is analysed therefore the prestressing force is obtained from TENDON:

$$P_0 = 3653\ kN$$

$$\sigma_{p,0} = 1281.755\ MPa$$

Calculating the prestressing losses due relaxation

According to EN 1992-1-1, 3.3.2 (5), the value of ρ_{1000} is expressed as a percentage ratio of the initial stress and is obtained for an initial stress equal to $0.7 \cdot f_p$, where f_p is the actual tensile strength of the prestressing steel samples. For design calculations, the characteristic tensile strength (f_{pk}) is used.

$$\mu = \frac{\sigma_{p0}}{f_{pk}} = \frac{1281.755}{1770} = 0.724155 \geq \mu_{min} = 0.55$$

EC2-1-1, 3.3.2 (7): Eq. 3.29

The relaxation loss may be obtained from the manufactures test certificates or defined as percentage ratio of the variation of the prestressing stress over the initial prestressing stress, should be determined by applying next expression:

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0.66 \cdot \rho_{1000} \cdot e^{9.1 \cdot \mu} \cdot \left(\frac{t}{1000}\right)^{0.75 \cdot (1-\mu)} \cdot 10^{-5}$$

$$t = 1000\ h \rightarrow \left(\frac{t}{1000}\right)^{0.75 \cdot (1-\mu)} = 1.00$$

EC2-1-1, 3.3.2 (6)

The values for ρ_{1000} can be either assumed equal to 8% for Class 1, **2.5% for Class 2**, and 4% for Class 3, or taken from the certificate.

For prestressing steel Y 1570/1770 $\rightarrow \rho_{1000} = 2.5\ \%$

The relaxation is checked for $\mu = 0.7$ and for $\mu = 0.72415$

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0.66 \cdot \rho_{1000} \cdot e^{9.1 \cdot 0.70} \cdot 10^{-5} \cdot 100 = 0.963695\ \%$$

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0.66 \cdot \rho_{1000} \cdot e^{9.1 \cdot 0.724155} \cdot 10^{-5} \cdot 100 = 1.200616\ \%$$

$\Delta\sigma_{pr} = \sigma_{pi} \cdot 1.200616\ \%$

$\Delta\sigma_{pr} = 1281.755 \cdot 1.200616\ \% = 15.388\ MPa$

AQB reduces the initial stress according to DIN 1045-1:

DIN 1045-1 8.7.3 (6)

$$\Delta P_{c+s+r} = A_p \frac{\epsilon_{cs} \cdot E_p + \Delta\sigma_{pr} + \dfrac{E_p}{E_{cm}} \cdot \phi(t,t_0) \cdot \sigma_{c,QP}}{1 + \dfrac{E_p}{E_{cm}} \cdot \dfrac{A_p}{A_c} \left(1 + \dfrac{A_c}{I_c} \cdot z_{cp}^2\right)[1 + 0.8 \cdot \phi(t,t_0)]}$$

DIN 1045-1 8.7.3 (6): ΔP_{c+s+r} time dependent losses of prestress

Creep and shrinkage is not taken into account therefore we have:

$\epsilon_{cs} \cdot E_p = 0$

$\dfrac{E_p}{E_{cm}} \cdot \phi(t,t_0) \cdot \sigma_{c,QP} = 0$

$1 + 0.8 \cdot \phi(t,t_0) = 1$

$$\Delta\sigma_{p,c+s+r} = \frac{15.388}{1 + \dfrac{195000}{34077} \cdot \dfrac{28.5 \cdot 10^{-4}}{0.9926} \cdot \left(1 + \dfrac{0.9926}{0.08214} \cdot 0.3901^2\right)}$$

$$\Delta\sigma_{p,c+s+r} = \frac{15.388}{1.046644} = 14.70\ MPa$$

Now we have:

$\sigma_{p0} = \sigma_{pg0} - 0.3 \cdot \Delta\sigma_{p,c+s+r}$

$\sigma_{p0} = 1281.755 - 0.3 \cdot 14.70 = 1277.345\ MPa$

DIN 1045-1 8.7.3 (6)

Now with the reduced stress the μ value is calculated again (iteration):

$$\mu = \frac{\sigma_{p0}}{f_{pk}} = \frac{1277.345}{1770} = 0.72166 \geq \mu_{min} = 0.55$$

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0.66 \cdot \rho_{1000} \cdot e^{9.1 \cdot 0.72166} \cdot 10^{-5} \cdot 100 = 1.1736\ \%$$

$\Delta\sigma_{pr} = 1281.755 \cdot 1.1736\% = 15.043\ MPa$

The total loss:

$\Delta\sigma_{pr,total} = \Delta\sigma_{pr,t} + \Delta\sigma_{pr,c}$

$\Delta\sigma_{pr,t} = \Delta\sigma_{pr} = 15.043\ MPa$

$\Delta\sigma_{pr,c} = (\epsilon_{k0} + \epsilon_{ky} \cdot z_{cp} - \epsilon_{kz} \cdot y_{cp}) \cdot E_p$

ϵ_{k0}, ϵ_{ky} and ϵ_{kz} - tendon strain

$\Delta\sigma_{pr,c} = (1.212 \cdot 10^{-6} + 5.673 \cdot 10^{-6} \cdot 0.39) \cdot 1.95 \cdot 10^8/1000$

$\Delta\sigma_{pr,c} = 0.667968\ MPa$

$\Delta\sigma_{pr,total} = 15.043 - 0.667968 = 14.37\ MPa$

$\Delta P_{pr} = \Delta\sigma_{pr,total} \cdot A_p = 14.37 \cdot 28.5 \cdot 10^{-4} = 40.9545\ kN$

Prestress force loss in [%]:

$$\Delta P_{pr}[\%] = \frac{\Delta P_{pr}}{P_0}$$

$$\Delta P_{pr}[\%] = \frac{40.9545}{3653} = 0.01120 \cdot 100\% = 1.12\ \%$$

5 Conclusion

This example shows the calculation of the time dependent losses due to relaxation. It has been shown that the reference solution and the AQB solution are in very good agreement.

6 Literature

[1] *EN 1992-1-1: Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings*. CEN. 2004.