SOFiSTiK

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Benchmark Example No. 19

Two-span Beam with Warping Torsion and Compressive Force

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VERiFiCATiON
BE19 Two-span Beam with Warping Torsion and Compressive Force

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Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview	
Element Type(s):	B3D
Analysis Type(s):	STAT, GNL
Procedure(s):	
Topic(s):	
Module(s):	ASE, DYNA
Input file(s):	two_span_beam.dat

1 Problem Description

The problem consists of a two-span beam, subjected to a large compressive axial force N_2 at its right end node, as well as a torsional moment M_t at the middle and an additional axial force N_1 in the middle of the right span, as shown in Fig. 1. The structure is examined for its torsional and warping behaviour.

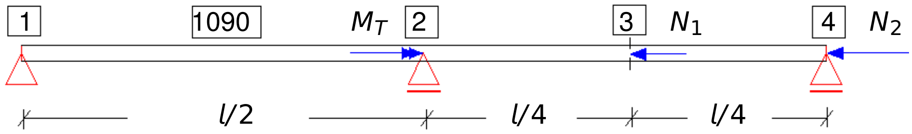

Figure 1: Problem Description

2 Reference Solution

While in first order theory, the axial force has no effect in the torsional deformations and moments, in second order torsional theory, the influence of the axial force in the rotation and twisting is considered. From the formulation of the equilibrium conditions at the twisted element, the torsional moment part M_{T3} results, which covers the contribution of the axial force in the total torsional moment. Therefore second order theory is utilised here, in order to account for the torsional effect of the axial force, as well as the warping torsion arising from the application of the torsional moment and the axial force at the intermediary nodes of the beam. The total torsional moment M_T is given as a sum of the different torsional parts, the primary, secondary and third respectively:

$$\sum M_x = M_T = M_{T1} + M_{T2} + M_{T3}, \tag{1}$$

where

$$M_{T1} = G\, I_T\, \phi', \tag{2}$$

$$M_{T2} = -E\, C_M\, \phi''', \tag{3}$$

$$M_{T3} = N\, i_p^2\, \phi', \tag{4}$$

and the warping moment

$$M_\omega = - E\,C_M\,\phi^{''}. \tag{5}$$

where G is the shear modulus, I_T the torsional moment of inertia, i_p the polar radius of gyration and EC_M the warping torsion stiffness. Introducing the above into Eq. 1 we have:

$$\left(G\,I_T\,+\,N\,i_p^2\right)\phi^{'} - E\,C_M\,\phi^{'''} = M_T\,=\sum M_x. \tag{6}$$

3 Model and Results

The properties of the model [1] are defined in Table 1. A standard steel material is used and an I-beam profile for the cross-section. A safety factor $\gamma_M = 1.1$ is used, which according to DIN 18800-2 it is applied both to the yield strength and the stiffness. At the supports the warping is not constrained. The cross-sectional properties, given in Table 1, are the values calculated by SOFiSTiK, matching the reference solution, except from the torsional moment I_T and the warping modulus C_M, which are modified to match the values of the reference example. This modification is done only for the sake of comparison and it has to be noted that the reference results [1] are computed with another finite element software, and not with respect to an analytical solution. The results are presented in Table 2, 3 and Fig 2. The double result values given for some nodes, e.g. 309/308, indicate the value left and right of the node respectively, and the exact result lies in between. When '—' is used, it indicates a change in the moment diagram.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$\gamma_M = 1.1$	$b = 180\,mm$, $h = 400\,mm$	$N_1 = 200\,kN$
$l = 6\,m$	$t_{web} = 10\,mm$, $t_{flange} = 14\,mm$	$N_2 = 1600\,kN$
$S\,355$	$I_y = 23071.6\,cm^4$, $I_z = 1363.9\,cm^4$	$M_t = 280\,kN\,cm$
	$C_M = 506900\,cm^6$	
	$I_T = 45.00\,cm^4$	

Table 2: Torsional Deformation Results

	Node 1		Node 2	
	SOF.	Ref.[1]	SOF.	Ref.[1]
$\phi\ [mrad]$	-	-	294	294
$\phi^{'}\ [rad/cm]$	1.525	1.52	-	-

Table 3: Torsional Moment Results

Node		M_T [$kNcm$]	M_{T1} [$kNcm$]	M_{T2} [$kNcm$]	M_{T3} [$kNcm$]	M_ω [kNm^2]
1	SOF.	121	505	382	−766	0
	Ref. [1]	121	505	382	−766	0
1090	SOF.	121	363	309/308	−550/−550	5.35
	Ref. [1]	121	363	308	−550	5.35
2	SOF.	121|−159	−9	118|−165	13/13	8.65
	Ref. [1]	121|−159	−9	117|−163	14	8.65
3	SOF.	−159	−364	−345|−285	551|490	4.70
	Ref. [1]	−159	−363	−346|−285	551|490	4.70
4	SOF.	−159	−487	−328	656	0
	Ref. [1]	−159	−487	−328	656	0

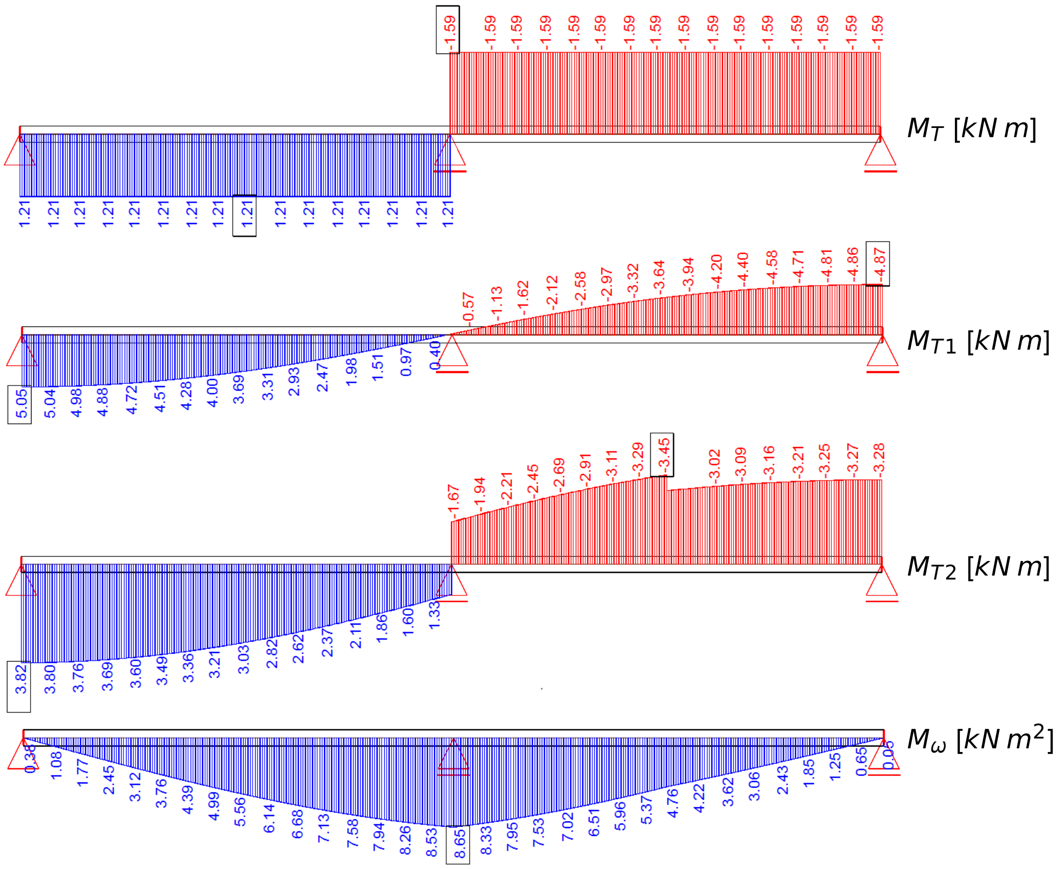

Figure 2: Results

In reference [1], except from the second order theory, the example is also analysed with respect to geometrically nonlinear torsional theory which accounts additionally for the large torsional deformations. This is done by introducing an additional torsional moment part, the helix torsional moment M_{TH}. The results of both analysis are compared, leading to the conclusion that second order theory lies almost always to the safe side.

4 Conclusion

This example examines the torsional behaviour of the beam and the different parts involved in the calculation of the total torsional moment. The results are reproduced accurately.

5 Literature

[1] V. Gensichen and G. Lumpe. *Anmerkungen zur linearen und nichtlinearen Torsionstheorie im Stahlbau*. Stahlbau Seminar 2012.