



Benchmark Example No. 22

## Tunneling - Ground Reaction Line

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**VERiFiCATiON**  
**BE22 Tunneling - Ground Reaction Line**

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The manual and the program have been thoroughly checked for errors. However, SOFiSTiK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.

**Front Cover**

Volkstheater, Munich Photo: Florian Schreiber

### Overview

<b>Element Type(s):</b>	C2D
<b>Analysis Type(s):</b>	STAT, MNL
<b>Procedure(s):</b>	LSTP
<b>Topic(s):</b>	SOIL
<b>Module(s):</b>	TALPA
<b>Input file(s):</b>	<a href="#">groundline_hoek.dat</a>

## 1 Problem Description

This problem consists of a cylindrical hole in an infinite medium, subjected to a hydrostatic in-situ state, as shown in Fig. 1. The material is assumed to be linearly elastic-perfectly plastic with a failure surface defined by the Mohr-Coulomb criterion and with zero volume change during plastic flow. The calculation of the ground reaction line is performed and compared to the analytical solution according to Hoek [1] [2].

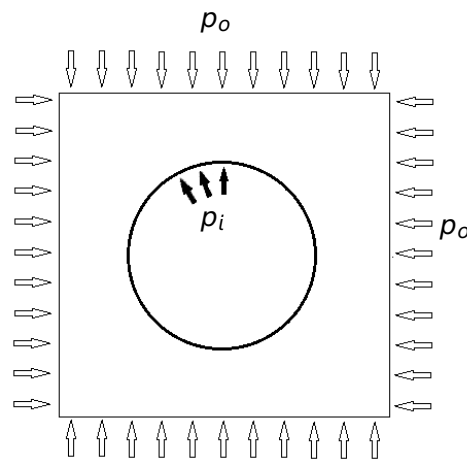


Figure 1: Problem Description

## 2 Reference Solution

The stability of deep underground excavations depends upon the strength of the rock mass surrounding the excavations and upon the stresses induced in this rock. These induced stresses are a function of the shape of the excavations and the in-situ stresses which existed before the creation of the excavations [1]. When tunnelling in rock, it should be examined how the rock mass, surrounding the tunnel, deforms and how the support system acts to control this deformation. In order to explore this effect, an analytical solution for a circular tunnel will be utilised, which is based on the assumption of a hydrostatic in-situ state. Furthermore, the surrounding rock mass is assumed to follow an elastic-perfectly-plastic material behaviour with zero volume change during plastic flow. Therefore the Mohr-Coulomb failure criterion is adopted, in order to model the progressive plastic failure of the rock mass surrounding the tunnel. The onset of plastic failure, is thus expressed as:

$$\sigma_1 = \sigma_{cm} + k\sigma_3, \quad (1)$$

where  $\sigma_1$  is the axial stress where failure occurs,  $\sigma_3$  the confining stress and  $\sigma_{cm}$  the uniaxial compress-

sive strength of the rock mass defined by:

$$\sigma_{cm} = \frac{2c \cos \phi}{1 - \sin \phi}. \quad (2)$$

The parameters  $c$  and  $\phi$  correspond to the cohesion and angle of friction of the rock mass, respectively. The tunnel behaviour on the other hand, is evaluated in terms of the internal support pressure. A circular tunnel of radius  $r_o$  subjected to hydrostatic stresses  $p_o$  and a uniform internal support pressure  $p_i$ , as shown in Fig. 2, is assumed.

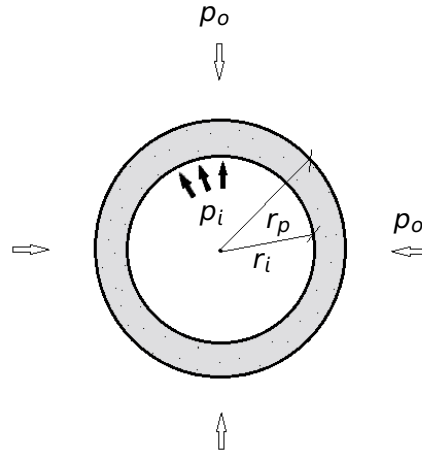


Figure 2: Plastic zone surrounding a circular tunnel

As a measure of failure, the critical support pressure  $p_{cr}$  is defined:

$$p_{cr} = \frac{2p_o - \sigma_{cm}}{1 + k}, \quad (3)$$

where  $k$  is the coefficient of passive earth pressure defined by:

$$k = \frac{1 + \sin \phi}{1 - \sin \phi}. \quad (4)$$

If the internal support pressure  $p_i$  is greater than  $p_{cr}$ , the behaviour of the surrounding rock mass remains elastic and the inward elastic displacement of the tunnel wall is:

$$u_{ie} = \frac{r_o (1 + \nu)}{E} (p_o - p_i), \quad (5)$$

where  $E$  is the Young's modulus and  $\nu$  the Poisson's ratio. If  $p_i$  is less than  $p_{cr}$ , failure occurs and the

total inward radial displacement of the walls of the tunnel becomes:

$$u_{ip} = \frac{r_o(1+\nu)}{E} \left[ 2(1+\nu)(p_o - p_{cr}) \left( \frac{r_p}{r_o} \right)^2 - (1-2\nu)(p_o - p_i) \right], \quad (6)$$

and the plastic zone around the tunnel forms with a radius  $r_p$  defined by:

$$r_p = r_o \left[ \frac{2(p_o(k-1) + \sigma_{cm})}{(1+k)((k-1)p_i + \sigma_{cm})} \right]^{\frac{1}{(k-1)}} \quad (7)$$

### 3 Model and Results

The properties of the model are defined in Table 1. The Mohr-Coulomb plasticity model is used for the modelling of the rock behaviour. The load is defined as a unit supporting pressure, uniform along the whole line of the circular hole, following the real curved geometry. The ground reaction line is calculated, which depicts the inward oriented deformation along the circumference of the opening that is to be expected in dependence of the acting support pressure.

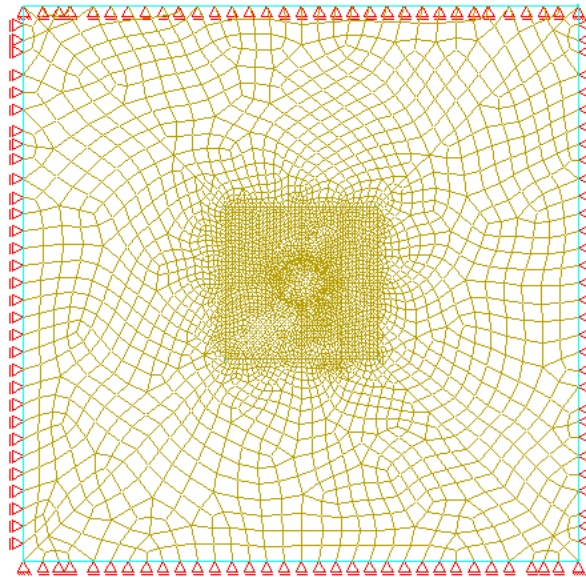


Figure 3: Finite Element Model

Table 1: Model Properties

Material Properties	Geometric Properties	Pressure Properties
$E = 5000000 \text{ kN/m}^2$	$r_o = 3.3 \text{ m}$	$P_o = 29700 \text{ kN/m}^2$
$\nu = 0.2$		$P_{i_{max}} = 7000 \text{ kN/m}^2$
$\gamma = 27 \text{ kN/m}^3$		$P_{cr} = 8133.744 \text{ kN/m}^2$
$\gamma_{buoyancy} = 17 \text{ kN/m}^3$		

Table 1: (continued)

Material Properties	Geometric Properties	Pressure Properties
$\phi = 39^\circ, \psi = 0^\circ$		
$c = 3700 \text{ kN/m}^2$		
$k = 4.395$		

The uniaxial compressive stress of the rock mass  $\sigma_{cm}$  is calculated at  $15514.423 \text{ kN/m}^2$  and the critical pressure  $p_{cr}$  is  $8133.744 \text{ kN/m}^2$ . The ground reaction line is presented in Fig. 4, as the curve of the inward radial displacement over the acting support pressure. It can be observed that the calculated values are in agreement with the analytical solution according to Hoek.

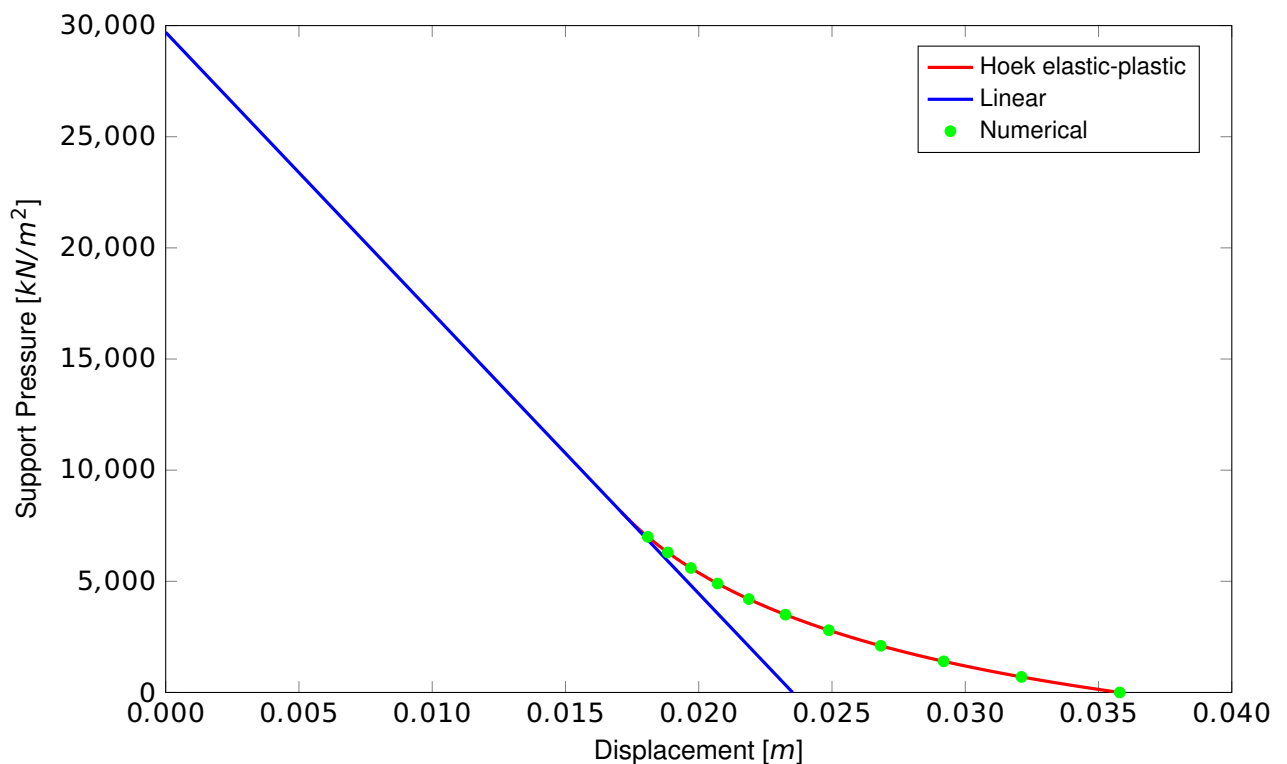


Figure 4: Ground Reaction Line

## 4 Conclusion

This example examines the tunnel deformation behaviour with respect to the acting support pressure. It has been shown that the behaviour of the tunnel in rock is captured accurately.

## 5 Literature

- [1] E. Hoek. *Practical Rock Engineering*. 2006.
- [2] E. Hoek, P.K. Kaiser, and W.F. Bawden. *Support of Underground Excavations in Hard Rock*. 1993.