



Benchmark Example No. 38

Calculation of Slope Stability by Phi-C Reduction

SOFiSTiK | 2023

VERiFiCATION
BE38 Calculation of Slope Stability by Phi-C Reduction

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The manual and the program have been thoroughly checked for errors. However, SOFiSTiK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.

Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview

Element Type(s):	C2D
Analysis Type(s):	STAT, MNL
Procedure(s):	LSTP, PHIC
Topic(s):	SOIL
Module(s):	TALPA
Input file(s):	slope_stability.dat

1 Problem Description

In this benchmark the stability of an embankment, as shown in Fig. 1, is calculated by means of a *phi*–*c* reduction. The factor of safety and its corresponding slip surface are verified.

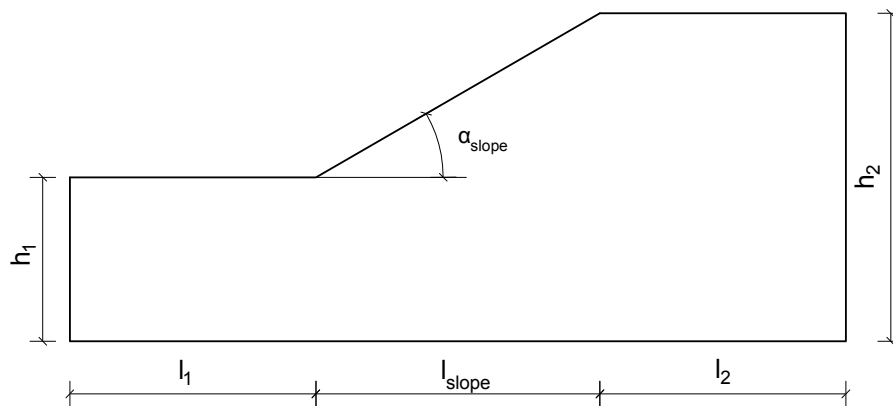


Figure 1: Problem Description

2 Reference Solution

The classical problem of slope stability analysis involves the investigation of the equilibrium of a mass of soil bounded below by an assumed potential slip surface and above by the surface of the slope. Forces and moments, tending to cause instability of the mass, are compared to those tending to resist instability. Most procedures assume a two-dimensional cross-section and plane strain conditions for analysis. Successive assumptions are made regarding the potential slip surface until the most critical surface, i.e. lowest factor of safety, is found. If the shear resistance of the soil along the slip surface exceeds that necessary to provide equilibrium, the mass is stable. If the shear resistance is insufficient, the mass is unstable. The stability of the mass depends on its weight, the external forces acting on it, the shear strengths and pore water pressures along the slip surface, and the strength of any internal reinforcement crossing potential slip surfaces. The factor of safety is defined with respect to the shear strength of the soil as the ratio of the available shear strength to the shear strength required for equilibrium [1]:

$$FS = \frac{\text{available shear strength}}{\text{equilibrium shear stress}} \quad (1)$$

The safety definition according to FELLENIUS is based on the investigation of the material's shear strength in the limit state of the system, i.e. the shear strength that leads to failure of the system.

Following this notion, in SOFiSTiK, the safety factors according to *phi* – *c* reduction are defined as the

ratio between available shear strength and the mobilized shear strength in the limit state of the system [2]:

$$\eta_{\phi} = \frac{\tan \phi_{inp}}{\tan \phi_{lim}} \quad (2)$$

$$\eta_c = \frac{c_{inp}}{c_{lim}} \quad (3)$$

where c is the cohesion and ϕ the friction angle. The ϕ – c reduction stability analysis is based on an incremental reduction of the shear strength adopting a synchronized increase of the safety factors $\eta = \eta_{\phi} = \eta_c$. The reached safety η at system failure represents the computational safety against stability failure.

The reference solution [3] is based on the finite element formulation of the upper- and lower-bound theorems of plasticity. Thus, the finite-element limit analysis (FELA) provides a good reference for the strength reduction method as it establishes upper and lower-bound estimates for the true stability limit.

3 Model and Results

The properties of the model [3] are presented in Table 1. The embankment has a slope height of 10 m and a slope angle of 30°. The initial stresses are generated using gravity loading. Then the embankment is subjected to the ϕ – c reduction. Plane strain conditions are assumed.

Table 1: Model Properties

Material Properties	Geometric Properties
$E = 20000 \text{ kN/m}^2, \nu = 0.3$	$h_1 = 20.0 \text{ m}$
$\gamma = 19 \text{ kN/m}^3$	$h_2 = 10.0 \text{ m}$
$\phi = 25^\circ, \psi = 25^\circ$	$l_1 = l_2 = 15.0 \text{ m}$
$c = 20 \text{ kN/m}^2$	$\alpha_{slope} = 30^\circ, l_{slope} = 17.321 \text{ m}$

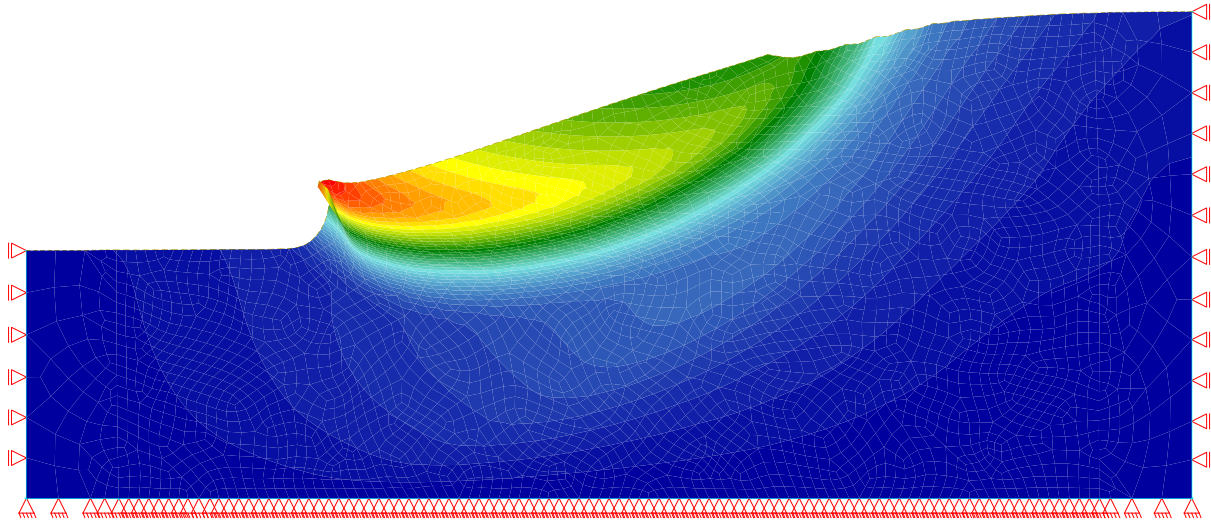


Figure 2: Nodal displacements for the factor of safety obtained with the $\phi - c$ reduction analysis

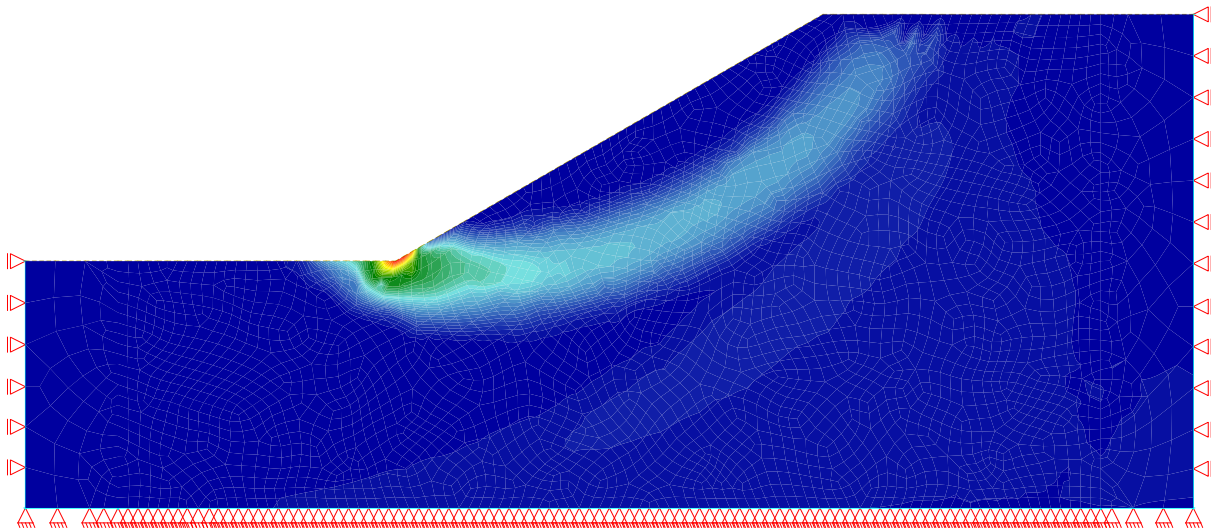


Figure 3: Deviatoric strain for the factor of safety obtained with the $\phi - c$ reduction analysis

Figure 2 presents the nodal displacement as a vector distribution for the factor of safety obtained with the $\phi - c$ reduction analysis. Furthermore, the corresponding plastic deviatoric strain is shown in Figure 3. The calculated factor of safety is compared with the reference solution [3] in Table 2, i.e. with the results from the lower-bound and upper-bound *finite element limit analysis* (FELA). Additionally, the calculated factor of safety from $\phi - c$ reduction analysis is plotted in Figure 4 as a function of the nodal displacement in x direction for the node at the top of the embankment slope.

Table 2: Factor of safety - calculated and reference values according to [3]

SOFISTiK FEM	FELA _{lower bound}	FELA _{upper bound}
2.00	1.97	2.01

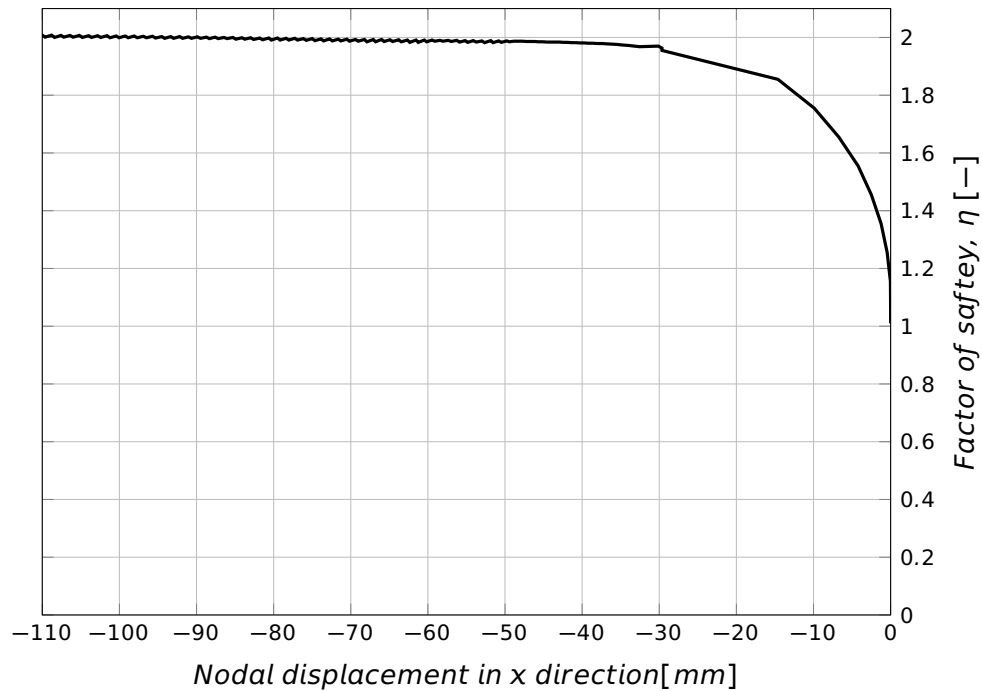


Figure 4: Factor of safety as a function of displacement in x direction for the node at the top of the embankment slope

4 Conclusion

This example verifies the stability of a soil mass and the determination of the factor of safety. The calculated factor of safety, which is obtained with the ϕ - c reduction method, is compared to the finite element limit analysis results and it is shown that the behavior of the model is captured accurately.

5 Literature

- [1] *USACE Engineering and Design: Slope Stability*. USACE. 2003.
- [2] *TALPA Manual: 2D Finite Elements in Geotechnical Engineering*. 2018-0. SOFiSTiK AG. Ober-schleißheim, Germany, 2017.
- [3] F. Tschuchnigg et al. "Comparison of finite-element limit analysis and strength reduction techniques". In: *Geotechnique* 65(4) (2015), pp. 249–257.