



Benchmark Example No. 39

Natural Frequencies of a Rectangular Plate

SOFiSTiK | 2023

VERiFiCATION
BE39 Natural Frequencies of a Rectangular Plate

VERiFiCATION Manual, Service Pack 2023-9 Build 42

Copyright © 2024 by SOFiSTiK AG, Nuremberg, Germany.

SOFiSTiK AG

HQ Nuremberg
Flataustraße 14
90411 Nürnberg
Germany

T +49 (0)911 39901-0
F +49(0)911 397904

Office Garching
Parkring 2
85748 Garching bei München
Germany

T +49 (0)89 315878-0
F +49 (0)89 315878-23

info@sofistik.com
www.sofistik.com

This manual is protected by copyright laws. No part of it may be translated, copied or reproduced, in any form or by any means, without written permission from SOFiSTiK AG. SOFiSTiK reserves the right to modify or to release new editions of this manual.

The manual and the program have been thoroughly checked for errors. However, SOFiSTiK does not claim that either one is completely error free. Errors and omissions are corrected as soon as they are detected.

The user of the program is solely responsible for the applications. We strongly encourage the user to test the correctness of all calculations at least by random sampling.

Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview

Element Type(s):	SH3D
Analysis Type(s):	DYN
Procedure(s):	EIGE
Topic(s):	
Module(s):	DYNA
Input file(s):	freq_plate.dat

1 Problem Description

This problem consists of a rectangular plate which is simply supported on all four sides, as shown in Fig. 1. The eigenfrequencies of the system are determined and compared to the exact reference solution for each case.

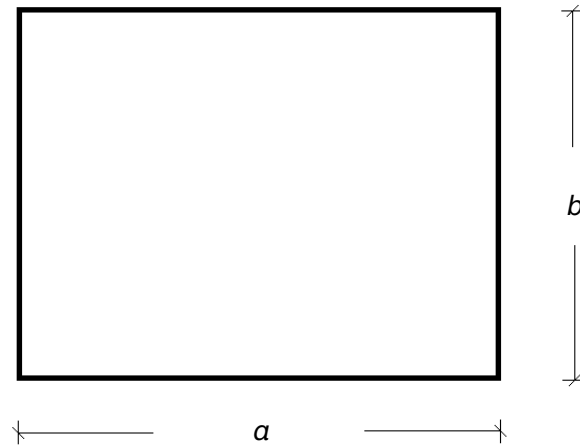


Figure 1: Problem Description

2 Reference Solution

The general formula to determine the eigenfrequencies of a simply-supported thin plate, consisting of a linear elastic homogeneous and isotropic material is given by [1], [2]

$$f_{m,n} = \frac{\lambda_{m,n}^2}{2\pi} \sqrt{\frac{gD}{\gamma h}} \quad (1)$$

where

$$\lambda_{m,n}^2 = \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \quad (2)$$

and D is the flexural rigidity of the plate

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (3)$$

Combining the above equations gives

$$f_{m,n} = \frac{\pi}{2a^2} \left(m^2 + n^2 \frac{a^2}{b^2} \right) \sqrt{\frac{g}{\gamma h} \frac{Eh^3}{12(1-\nu^2)}} \quad (4)$$

where a , b the dimensions of the plate, h the thickness and $\gamma h/g$ the mass of the plate per unit area. The values of $\lambda_{m,n}^2$ for the first five combinations of m , n are given in Table 1 for a simply-supported plate.

Table 1: Dimensionless parameter $\lambda_{m,n}^2$

m	n	$\lambda_{m,n}^2$	Mode number
1	1	32.08	1
2	1	61.69	2
1	2	98.70	3
3	1	111.03	4
2	2	128.30	5

3 Model and Results

The properties of the model are defined in Table 2 and the resulted eigenfrequencies are given in Table 3. The corresponding eigenforms are presented in Fig. 2.

Table 2: Model Properties

Material Properties	Geometric Properties
$E = 30000 \text{ MPa}$	$a = 4.5 \text{ m}$
$\gamma = 80 \text{ kN/m}^3$	$b = 3.0 \text{ m}$
$\nu = 0.3$	$h = 0.02 \text{ m}$

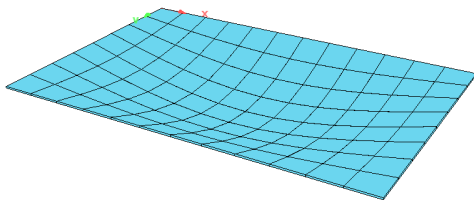
Table 3: Results

Eigenfrequency Number	SOF. [Hz]	Ref. [Hz]	$ e_r $ [%]
1	2.941	2.955	0.476

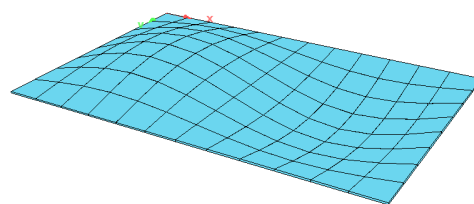
Table 3: (continued)

Eigenfrequency Number	SOF. [Hz]	Ref. [Hz]	$ e_r $ [%]
2	5.623	5.682	1.047
3	9.200	9.091	1.197
4	10.206	10.228	0.214
5	11.706	11.819	0.954

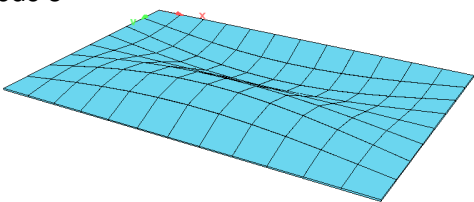
Mode 1



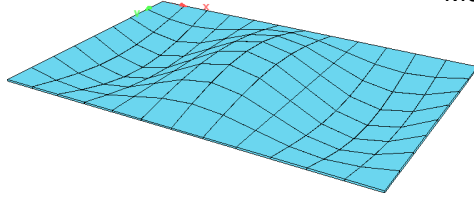
Mode 2



Mode 3



Mode 4



Mode 5

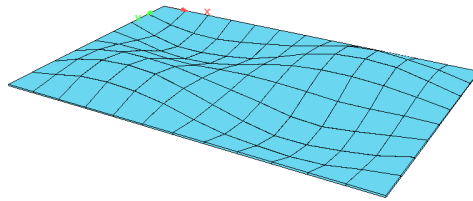


Figure 2: Eigenforms

4 Conclusion

The purpose of this example is to verify the eigenvalue determination of plate structures modelled with plane elements. It has been shown that the eigenfrequencies for a simply-supported thin rectangular plate are calculated accurately.

5 Literature

- [1] S. Timoshenko. *Vibration Problems in Engineering*. 2nd. D. Van Nostrand Co., Inc., 1937.
- [2] Schneider. *Bautabellen für Ingenieure*. 19th. Werner Verlag, 2010.