Benchmark Example No. 39

Natural Frequencies of a Rectangular Plate

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VERiFiCATiON
BE39 Natural Frequencies of a Rectangular Plate

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Front Cover

Volkstheater, Munich Photo: Florian Schreiber

Overview	
Element Type(s):	SH3D
Analysis Type(s):	DYN
Procedure(s):	EIGE
Topic(s):	
Module(s):	DYNA
Input file(s):	freq_plate.dat

1 Problem Description

This problem consists of a rectangular plate which is simply supported on all four sides, as shown in Fig. 1. The eigenfrequencies of the system are determined and compared to the exact reference solution for each case.

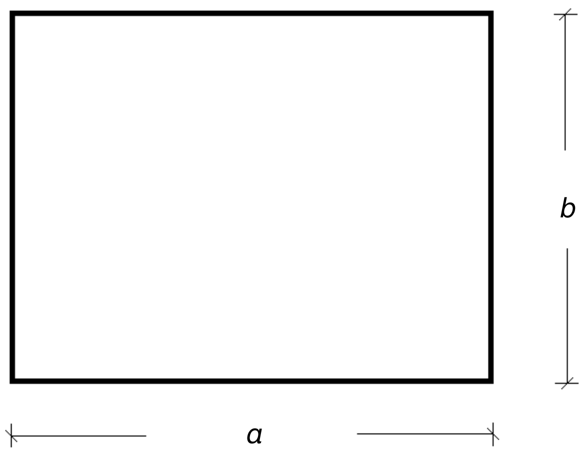

Figure 1: Problem Description

2 Reference Solution

The general formula to determine the eigenfrequencies of a simply-supported thin plate, consisting of a linear elastic homogeneous and isotropic material is given by [1], [2]

$$f_{m,n} = \frac{{\lambda_{m,n}}^2}{2\pi}\sqrt{\frac{gD}{\gamma h}} \tag{1}$$

where

$$ {\lambda_{m,n}}^2 = \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \tag{2}$$

and D is the flexural rigidity of the plate

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{3}$$

Combining the above equations gives

$$f_{m,n} = \frac{\pi}{2a^2}\left(m^2 + n^2\frac{a^2}{b^2}\right)\sqrt{\frac{g}{\gamma h}\frac{Eh^3}{12(1-\nu^2)}} \tag{4}$$

where a, b the dimensions of the plate, h the thickness and $\gamma h/g$ the mass of the plate per unit area. The values of ${\lambda_{m,n}}^2$ for the first five combinations of m, n are given in Table 1 for a simply-supported plate.

Table 1: Dimensionless parameter ${\lambda_{m,n}}^2$

m	n	${\lambda_{m,n}}^2$	Mode number
1	1	32.08	1
2	1	61.69	2
1	2	98.70	3
3	1	111.03	4
2	2	128.30	5

3 Model and Results

The properties of the model are defined in Table 2 and the resulted eigenfrequencies are given in Table 3. The corresponding eigenforms are presented in Fig. 2.

Table 2: Model Properties

Material Properties	Geometric Properties
$E = 30000\ MPa$	$a = 4.5\ m$
$\gamma = 80\ kN/m^3$	$b = 3.0\ m$
$\nu = 0.3$	$h = 0.02\ m$

Table 3: Results

Eigenfrequency Number	SOF. $[Hz]$	Ref. $[Hz]$	$\lvert e_r\rvert$ [%]
1	2.941	2.955	0.476

Table 3: (continued)

Eigenfrequency Number	SOF. $[Hz]$	Ref. $[Hz]$	$\lvert e_r \rvert$ [%]
2	5.623	5.682	1.047
3	9.200	9.091	1.197
4	10.206	10.228	0.214
5	11.706	11.819	0.954

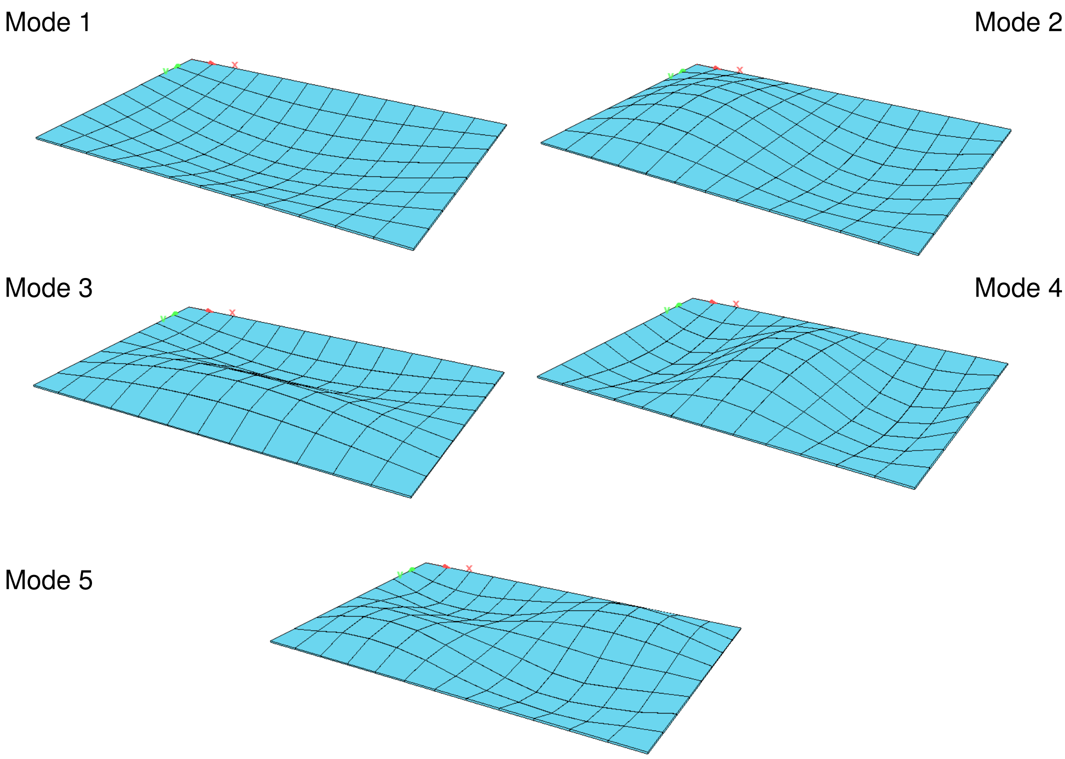

Figure 2: Eigenforms

4 Conclusion

The purpose of this example is to verify the eigenvalue determination of plate structures modelled with plane elements. It has been shown that the eigenfrequencies for a simply-supported thin rectangular plate are calculated accurately.

5 Literature

[1] S. Timoshenko. *Vibration Problems in Engineering*. 2nd. D. Van Nostrand Co., Inc., 1937.
[2] Schneider. *Bautabellen für Ingenieure*. 19th. Werner Verlag, 2010.