SOFiSTiK

Benchmark Example No. 14

Buckling of a Bar with Hinged Ends II

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VERiFiCATiON
BE14 Buckling of a Bar with Hinged Ends II

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Front Cover

6th Street Viaduct, Los Angeles Photo: Tobias Petschke

Overview	
Element Type(s):	SH3D
Analysis Type(s):	STAT, GNL
Procedure(s):	STAB
Topic(s):	
Module(s):	ASE
Input file(s):	buckling_bar_quad.dat

1 Problem Description

Benchmark Example 13 is tested here for QUAD plane elements. The problem consists of an axially loaded long slender bar of length l with hinged ends, as shown in Fig. 1. Determine the critical buckling load [1].

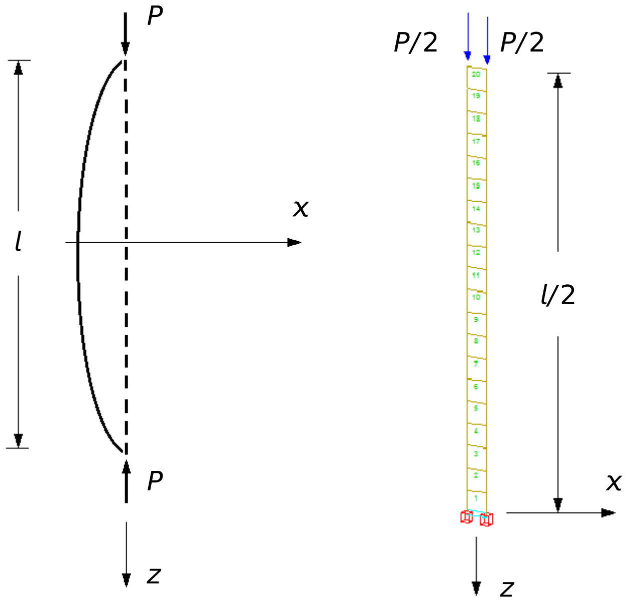

Figure 1: Problem Description

2 Reference Solution

The problem of lateral buckling of bars is presented at Benchmark Example 13. For a prismatic bar, the critical load is [2]:

$$P_{cr} = \frac{\pi^2 EI}{(\beta l)^2}. \tag{1}$$

From the above equation it is evident, that the critical load does not depend upon the strength of the material but only upon the dimensions of the structure and the modulus of elasticity of the material. Two equal slender axially compressed bars, will buckle at the same compressive force, if they consist of the same flexural rigidity and material with the same Young's modulus.

3 Model and Results

Only the upper half of the bar is modelled because of symmetry (Fig. 1). The boundary conditions thus become free-fixed for the half symmetry model. A total of 20 elements are used to capture the buckling mode. The properties of the model are defined in Table 1.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$E = 300\,MPa$	$l = 20\,m$, $h = 0.5\,m$	$P = 1\,kN$
	$A = 0.25\,m^2$	$P_x << 1\,kN$
	$t = 0.5\,m$	
	$I = 5.20833 \times 10^{-3}\,m^4$	
	$\beta = 2$, free-fixed ends	

A buckling eigenvalue determination is performed where the critical load factor is calculated. Result of the eigenvalue calculation are presented in Table 2. The reference value of the critical load for Benchmark Example 13 and 14 is calculated the same, since the properties of the two models are equivalent, as explained in Section 2.

Table 2: Results

Solver	P_{cr} $[kN]$	Ref.	$\lvert e_r \rvert$ [%]
BUCK - Simultaneous vector iteration	38.539	38.553	0.0379

4 Conclusion

This example presents the buckling of slender bars. It has been shown that the buckling properties of the bar are accurately captured also with QUAD elements.

5 Literature

[1] *Verification Manual for the Mechanical APLD Application, Release 12.0*. Ansys, Inc. 2009.
[2] S. Timoshenko. *Strength of Materials, Part II, Advanced Theory and Problems*. 2nd. D. Van Nostrand Co., Inc., 1940.