Benchmark Example No. 9

Design of a Rectangular CS for Shear and Torsion

SOFiSTiK | 2024

VERiFiCATiON
DCE-EN9 Design of a Rectangular CS for Shear and Torsion

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Front Cover

6th Street Viaduct, Los Angeles Photo: Tobias Petschke

Overview	
Design Code Family(s):	DIN
Design Code(s):	DIN EN 1992-1-1
Module(s):	AQB
Input file(s):	rectangular_shear_torsion.dat

1 Problem Description

The problem consists of a rectangular section, symmetrically reinforced for bending, as shown in Fig. 1. The cross-section is designed for shear force V_{Ed} and torsion T_{Ed} and the required shear and torsion reinforcement is determined.

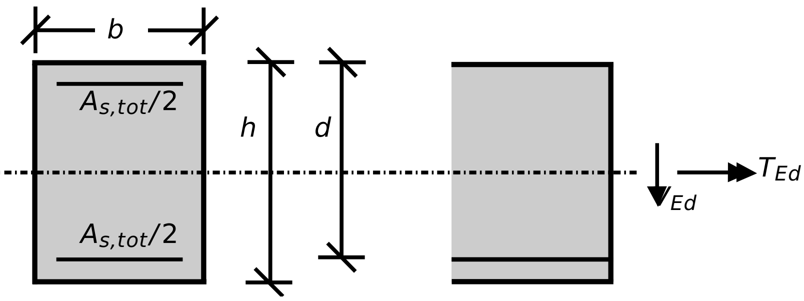

Figure 1: Problem Description

2 Reference Solution

This example is concerned with the design of sections for ULS, subject to shear force and torsion. The content of this problem is covered by the following parts of DIN EN 1992-1-1:2004 [1] [2]:

- Design stress-strain curves for concrete and reinforcement (Section 3.1.7, 3.2.7)
- Guidelines for shear (Section 6.2) and torsion design (Section 6.3)
- Reinforcement (Section 9.2.2, 9.2.3)

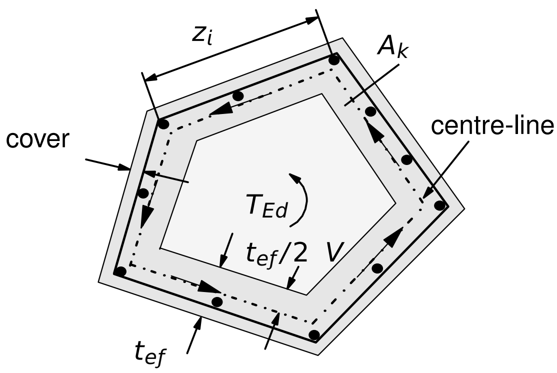

Figure 2: Torsion Reinforced Members

The design stress-strain diagram for reinforcing steel considered in this example, consists of an inclined top branch, as presented in Fig. 3 and as defined in DIN EN 1992-1-1:2004 [1] (Section 3.2.7).

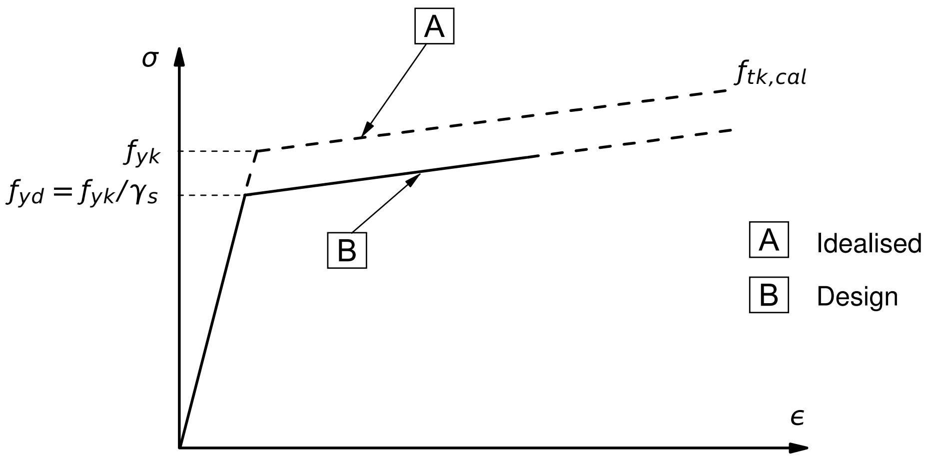

Figure 3: Idealised and Design Stress-Strain Diagram for Reinforcing Steel

3 Model and Results

The rectangular cross-section, with properties as defined in Table 1, is to be designed, with respect to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], under shear force of $175.0\ kN$ and torsional moment $35\ kNm$. The reference calculation steps [3] are presented below and the results are given in Table 2.

Table 1: Model Properties

Material Properties	Geometric Properties	Loading
$C\ 35/45$	$h = 70.0\ cm$	$V_{Ed} = 175.0\ kN$
$B\ 500A$	$b = 30\ cm$	$T_{Ed} = 35.0\ kNm$
	$d = 65.0\ cm$	
	$A_{s,tot} = 26.8\ cm^2$	

Table 2: Results

	SOF	Ref.
$A_{sw} / s_w\ (T)\ \ [cm^2/m]$	3.35	3.35
$A_{sl}\ (T)\ \ [cm^2]$	5.37	5.37
$A_{sw,total} / s\ \ [cm^2/m]$	13.60	13.59
$V_{Rd,c}\ [kN]$	110.13	113.12
$V_{Rd,max}\ [kN]$	1303.05	1303.03
$T_{Rd,max}\ [kNm]$	124.95	124.95

4 Design Process [1]

Design with respect to DIN EN 1992-1-1:2004 (NA) [1] [2]:[2]

Material:

Concrete: $\gamma_c = 1.50$

(NA) 2.4.2.4: (1), Tab. 2.1DE: Partial factors for materials

Steel: $\gamma_s = 1.15$

$f_{ck} = 30\ MPa$

Tab. 3.1: Strength for concrete

$f_{cd} = \alpha_{cc} \cdot f_{ck} / \gamma_c = 0.85 \cdot 35/1.5 = 19.833\ MPa$

(NDP) 3.1.6: (1)P, Eq. (3.15): $\alpha_{cc} = 0.85$ considering long term effects

$f_{yk} = 500\ MPa$

(NDP) 3.2.2: (3)P: yield strength $f_{yk} = 500\ MPa$

$f_{yd} = f_{yk} / \gamma_s = 500/1.15 = 434.78\ MPa$

3.2.7: (2), Fig. 3.8

Design Load:

$V_{Ed} = 175.0\ kN,\ T_{Ed} = 35.0\ kN$

$$T_{Ed} \leq \frac{V_{Ed} \cdot b_w}{4.5}$$

$$35 > \frac{175 \cdot 0.3}{4.5} = 11.66$$

(NDP) 6.3.2 (5): Eq. (NA.6.31.1): For approximately rectangular solid sections no shear and torsion reinforcement is required except from the minimum reinforcement, provided that the condition is satisfied

→ Eq. NA.6.31.1 is not fulfilled

$$V_{Ed} \cdot \left[1 + \frac{4.5 \cdot T_{Ed}}{V_{Ed} \cdot b_w}\right] \leq V_{Rd,c}$$

and (NDP) 6.3.2 (5): Eq. (NA.6.31.2): When the condition equation is not fulfilled then shear and torsion design has to be reverified

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.12 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

(NDP) 6.2.2 (1): Eq. 6.2a: Design value for shear resistance $V_{Rd,c}$ for members not requiring design shear reinforcement

$$C_{Rd,c} = 0.15/\gamma_c = 0.1$$

(NDP) 6.2.2 (1): $C_{Rd,c} = 0.15/\gamma_c$

$$k = 1 + \sqrt{\frac{200}{d}} < 2.0$$

$$k = 1 + \sqrt{\frac{200}{650}} = 1.5547 < 2.0$$

$$\rho_1 = \frac{A_{sl}}{b_w \cdot d}$$

$$\rho_1 = \frac{26.8 \cdot 100^{-2}}{0.3 \cdot 0.65} = 0.0137 < 0.02$$

$$V_{Rd,c} = \left[0.1 \cdot 1.5547 \cdot (100 \cdot 0.0137 \cdot 35)^{1/3} + 0\right] \cdot 0.3 \cdot 0.65$$

$$V_{Rd,c} = 0.11013\ MN = 110.13\ kN$$

$$175 \cdot \left[1 + \frac{4.5 \cdot 35}{175 \cdot 0.3}\right] = 700 > 111.74$$

[1] The tools used in the design process are based on steel stress-strain diagrams, as defined in [1] 3.2.7:(2), Fig. 3.8, which can be seen in Fig. 3.

[2] The sections mentioned in the margins refer to DIN EN 1992-1-1:2004 (German National Annex) [1], [2], unless otherwise specified.

$\rightarrow$ requirement of Eq. NA.6.31.2 is not met

Torsional reinforcement

6.3.1 (3): Solid sections may be modelled by equivalent thin-walled sections (Fig. 2)
A_k: area enclosed by the centre-line

$$t_{eff,1} = t_{eff,2} = 2 \cdot 50 = 100\ mm \ \ (s_o = s_u = s_s = 50\ mm)$$

$$A_k = (h - s_u - s_o) \cdot (b_w - t_{eff,1}) = 100\ mm$$

$$A_k = (700 - 50 - 50) \cdot (300 - 100) = 120000\ mm^2 = 0.12\ m^2$$

u_k: circumference of area A_k

$$u_k = 2 \cdot [(700 - 50 - 50) + (300 - 100)] = 1600\ mm = 1.6\ m$$

6.3.2 (2): The effects of torsion and shear may be superimposed, assuming the same value for θ

Simplifying, the reinforcement for torsion may be determined alone under the assumption of $\cot\theta = 1.0$, $\theta = 45°$ and be added to the independently calculated shear force reinforcement.

(NDP) 6.3.2 (3): Eq. (NA.6.28.1)

$$A_{sw,requ} \,/\, s_w = T_{Ed} \cdot \tan\theta \,/\, (f_{yd} \cdot 2A_k)$$

$$A_{sw,requ} \,/\, s_w = 350 \cdot 1.0 \,/\, (435 \cdot 2 \cdot 0.12) = 3.35\ cm^2/m\ (T)$$

6.3.2 (3): Eq. 6.28

$$A_{sl,requ} = T_{Ed} \cdot u_k \cdot \cot\theta \,/\, (f_{yd} \cdot 2A_k) = 5.37\ cm^2\ (T)$$

Torsional resistance moment

6.3.2 (4): Eq. 6.30
(NDP): $\nu = 0.525$

$$T_{Rd,max} = 2 \cdot \nu \cdot f_{cd} \cdot A_k \cdot t_{eff,i} \cdot \sin\theta \cdot \cos\theta$$

$\sin\theta \cdot \cos\theta = 0.5$ since $\theta = 45°$

$$T_{Rd,max} = 124.95\ kNm$$

Check of the concrete compressive strut bearing capacity for the load combination of shear force and torsion

The maximum resistance of a member subjected to torsion and shear is limited by the capacity of the concrete struts. The following condition should be satisfied:

(NDP) 6.3.2 (4): Eq. (NA.6.29.1) for solid cross-sections

$$\left[\frac{T_{Ed}}{T_{Rd,max}}\right]^2 + \left[\frac{V_{Ed}}{V_{Rd,max}}\right]^2 \leq 1.0$$

For the T+V utilization, SOFiSTiK uses:

$$\sqrt{\left[\frac{T_{Ed}}{T_{Rd,max}}\right]^2 + \left[\frac{V_{Ed}}{V_{Rd,max}}\right]^2} \leq 1.0$$

(NDP) 6.2.3 (1): Inner lever arm z

$$z = \max\left\{d - c_{V,l} - 30\ mm;\ d - 2\ c_{V,l}\right\}$$

s_o: offset of reinforcement
D_o: bar diameter

$$c_{V,l} = s_o - D_o/2 = 50 - 28/2 = 36\ mm$$

$$z = \max\{584;\ 578\} = 584\ mm$$

(NDP) 6.2.3 (3): Eq. 6.9
Maximum shear force $V_{Rd,max}$ occurs for $\theta = 45°$: $\cot\theta = \tan\theta = 1$
(NDP) 6.2.3 (3): $\nu_1 = 0.75 \cdot \nu_2 = 0.75$, $\nu_2 = 1$ for $\leq C50/60$

$$V_{Rd,max} = b_w \cdot z \cdot \nu_1 \cdot f_{cd} \,/\, (\cot\theta + \tan\theta)$$

$$V_{Rd,max} = 0.3 \cdot 0.584 \cdot 0.75 \cdot 19.833 \,/\, (1 + 1) = 1303.03\ kN$$

$$\left[\frac{35}{124.95}\right]^2 + \left[\frac{175}{1303.04}\right]^2 = 0.0965 < 1$$

Shear reinforcement

$A_{sw,requ} \,/\, s = V_{Ed} \,/\, (f_{ywd} \cdot z \cdot \cot\theta) = 6.89\ cm^2/m$

6.2.3 (3): Eq. 6.8
$f_{ywd} = f_{yk}/\gamma_s = 435\ MPa$

Total required reinforcement

Required torsional reinforcement:

$2 \cdot A_{sw}/s_w = 2 \cdot 3.35 = 6.7\ cm^2/m$ (double-shear connection)

Total reinforcement: $A_{sw,total}/s = 6.7 + 6.89 = 13.59\ cm^2/m$

Check the maximum allowable compressive stress

$$\nu_{cv} = 0.750 \cdot \eta_1 \cdot min(1.0, 1.1 - \frac{f_{ck}}{500})$$
$$= 0.750 \cdot 1.00 \cdot min(1.0, 1.1 - \frac{35}{500})$$
$$= 0.750$$

$$\nu_{cv} = 0.525 \cdot \eta_1 \cdot min(1.0, 1.1 - \frac{f_{ck}}{500})$$
$$= 0.525 \cdot 1.00 \cdot min(1.0, 1.1 - \frac{35}{500})$$
$$= 0.525$$

$$\sigma_{cv} = \nu_{cv} \cdot f_{cd}$$
$$= 0.75 \cdot 19.83$$
$$= 14.88$$

$$\sigma_{ct} = \nu_{ct} \cdot f_{cd}$$
$$= 0.525 \cdot 19.83$$
$$= 10.41$$

$\sigma_{II} < \sigma_{c,v+t}$

$-4.91\ MPa < -14.88\ MPa \rightarrow$ **OK**

5 Conclusion

This example shows the calculation of the required reinforcement for a rectangular beam cross-section under shear and torsion. It has been shown that the results are reproduced with excellent accuracy.

6 Literature

[1] *DIN EN 1992-1-1/NA: Eurocode 2: Design of concrete structures, Part 1-1/NA: General rules and rules for buildings - German version EN 1992-1-1:2005 (D), Nationaler Anhang Deutschland - Stand Februar 2010*. CEN. 2010.

[2] F. Fingerloos, J. Hegger, and K. Zilch. *DIN EN 1992-1-1 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken - Teil 1-1: Allgemeine Bemessungsregeln und Regeln für den Hochbau*. BVPI, DBV, ISB, VBI. Ernst & Sohn, Beuth, 2012.

[3] *Beispiele zur Bemessung nach Eurocode 2 - Band 1: Hochbau*. Ernst & Sohn. Deutschen Beton- und Bautechnik-Verein E.V. 2011.